Implementation of moving band solver for finite element analysis of electrical machines (repository copy)

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IMPLEMENTATION OF A MOVING BAND SOLVER FOR FINITE ELEMENT ANALYSIS OF ELECTRICAL MACHINES.

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Abstract: The simulation of magnetically geared electrical machines using the finite element method is an especially demanding task when movement has to be considered. Several methods that facilitate movement exist and the most prominent ones are described in this paper. Based on the characteristics of these methods, the moving band is selected as the most appropriate for the simulation of magnetically geared machines. This method is implemented in an in-house finite element package and its performance is evaluated using two case studies. The accuracy as well as the computational efficiency of the moving band technique is compared to that of the air-gap element. For magnetically geared machines, the moving band is the preferred choice because of its low computational cost and acceptable accuracy.

Key words: moving band, air-gap element, finite element analysis, electrical machines

1. INTRODUCTION

Magnetically geared machines (MGMs) are a new class of electrical machine that integrates a conventional permanent magnet machine with a concentric magnetic gear. These machines are worth considering because of the exceptionally high torque density that they offer. Analyzing these machines using the finite element method is very demanding for two reasons: Firstly, the lack of periodicity often necessitates modeling of the full machine. Secondly, the machines have multiple air-gaps (typically two or three) which increase the problem complexity when movement has to be considered. An example of such a magnetically geared machine with an inner stator is shown in Figure 1.

In this paper, several different methods that facilitate movement in finite element meshes are briefly reviewed considering the requirements for simulating magnetically geared permanent magnet machines. Two of these methods, implemented in an in-house finite element code, are evaluated using two case studies which highlight the relative merits of each method.

2. MOVEMENT FACILITATING TECHNIQUES

The main methods used to facilitate movement in finite element analysis of electrical machines are the air-gap element (AGE), Lagrange multiplier method (LMM), mortar element method (MEM) and the moving band (MB). In this section, these methods will be reviewed briefly.

2.1 Air-gap element

The air-gap element [1] is a technique whereby the field in the entire air-gap region is calculated analytically using
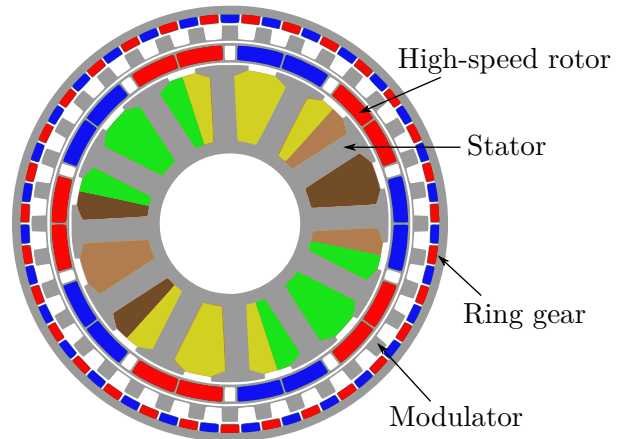

Figure 1: An example of a magnetically geared permanent magnet machine with an inner stator.

a Fourier series expansion of the vector potential. The field representation must satisfy the boundary conditions derived from the adjoining meshed regions. The vector potential in the air-gap takes the form

$$A(x,y) = \sum_i \alpha_i(x,y) u_i \qquad (1)$$

where the α_i fulfill the role of a shape function and the u_i are the nodal values of the vector potential on the AGE boundary. Movement is accomplished by a simple recalculation of the α_i without any modification to the mesh structure, resulting in simple and efficient time-stepping [2].

Considering (1), it is clear that all the air-gap nodes are connected and thus, this method results in a dense block appearing in the final system matrix (see Figure 10) which can have a drastic impact on the computational time

required to obtain a solution. An advantages of the air-gap element technique is that the results can be very accurate because of the high order of the field representation in the air-gap region. Also, the Fourier series representation of the vector potential can be used directly to calculate the torque as described in [3].

2.2 Lagrange multiplier method

The sliding surface technique was first proposed in [4]. The idea is to split the model into two separate domains, Ω_a and Ω_b, and to ensure the continuity of the vector potential across the domain boundaries, an additional constraint

$$I_\lambda = \int_\Gamma \lambda(A_a - A_b)\mathrm{d}\Gamma = 0 \qquad (2)$$

is added to the standard finite element formulation. The solution to the coupled problem is obtained by minimizing

$$I(A_a, A_b, \lambda) = I_a + I_b + I_\lambda \qquad (3)$$

with respect to the vector potentials of the two domains, A_a and A_b, and the Lagrange multipliers, λ. I_a and I_b are standard energy functionals,

$$I_a = \frac{1}{2} \int_{\Omega_a} \frac{1}{\mu} \left(\left[\frac{\mathrm{d}A_a}{\mathrm{d}x}\right]^2 + \left[\frac{\mathrm{d}A_a}{\mathrm{d}y}\right]^2 \right) \mathrm{d}\Omega_a \qquad (4)$$

$$I_b = \frac{1}{2} \int_{\Omega_b} \frac{1}{\mu} \left(\left[\frac{\mathrm{d}A_b}{\mathrm{d}x}\right]^2 + \left[\frac{\mathrm{d}A_b}{\mathrm{d}y}\right]^2 \right) \mathrm{d}\Omega_b \qquad (5)$$

The method also has the advantage that the structure of the mesh is maintained when moving, in other words, no remeshing is required. On the other hand, the method increases the system dimension by introducing an extra set of variables, the Lagrange multipliers.

2.3 Mortar element method

The mortar element method and the Lagrange multiplier method produce similar results [5], and can both be considered sliding surface techniques. In contrast to the LMM, the MEM deals with the interface between the two domains by considering one as the master and the other as the slave. The variables on the slave interface are functions of the variables on the master interface and so they can be eliminated. Thus, the methods differ in terms of the characteristics of the system matrices they produce. According to [5], the MEM produces a positive definite matrix whereas the LMM does not. This means that MEM systems can be solved by the Incomplete Choleski Conjugate Gradient (ICCG) method but LMM systems can not. On the other hand, MEM matrices have more nonzero entries than LMM matrices. It is concluded that the performance of MEM used together with ICCG quickly overtakes that of LMM with Gaussian elimination as the system dimension increases.

2.4 Moving band

This technique was first proposed in [6]. It has several advantages over the other techniques mentioned thus far. It employs no special elements or coupling techniques, it does not generated any dense blocks in the system matrix and it does not increase the system dimension. For these reasons, the moving band technique should be superior in terms of computational speed. However, there are difficulties with this method as well: Remeshing the air-gap region is inevitable and thus the numbering as well as the amount of nodes in the mesh does not stay constant. For this reason, the conditioning of the system matrix is not maintained and preconditioning routines must be rerun when the mesh changes. Also, because the elements in the air-gap are geometrically distorted to accommodate arbitrary movement, the results obtained using this method often have an oscillating error component.

2.5 General considerations

It is noted that the importance of having a sparse system matrix with a small profile depends on the method used to solve the system equation. In this study, a direct method (Lower-Upper factorisation) which is sensitive to the profile was used. Conjugate gradient methods may be less sensitive to the profile and could be considered in future.

Based on the above considerations, the moving band technique was selected as the most appropriate technique for modeling MGMs.

3. IMPLEMENTATION

In this section, the implementation of the moving band technique in an in-house Fortran finite element package call *SEMFEM* will be described.

A flowchart describing the working of the moving band solver is depicted in Figure 2. As shown in the figure, the first part of the solver consists of a single process which detects the number of air-gaps in the model and sets up the master data structures. The second part of the algorithm makes use of parallel processes where the number of time steps to be solved is divided among the number of threads. The parallelization is achieved using *OpenMP* [7]. Using this procedure, a significant speed-up can be achieved on multicore processors.

Two primary data structures are used in the solver. The first one stores the general mesh information, the second stores information on the moving bands. For every time-step, the bands have to be shifted which results in distortion of the elements and possibly reconnection of the air-gap nodes to avoid badly shaped elements. In periodic models where the full machine is not simulated, additional nodes also need to be added in order to maintain correct boundary conditions.
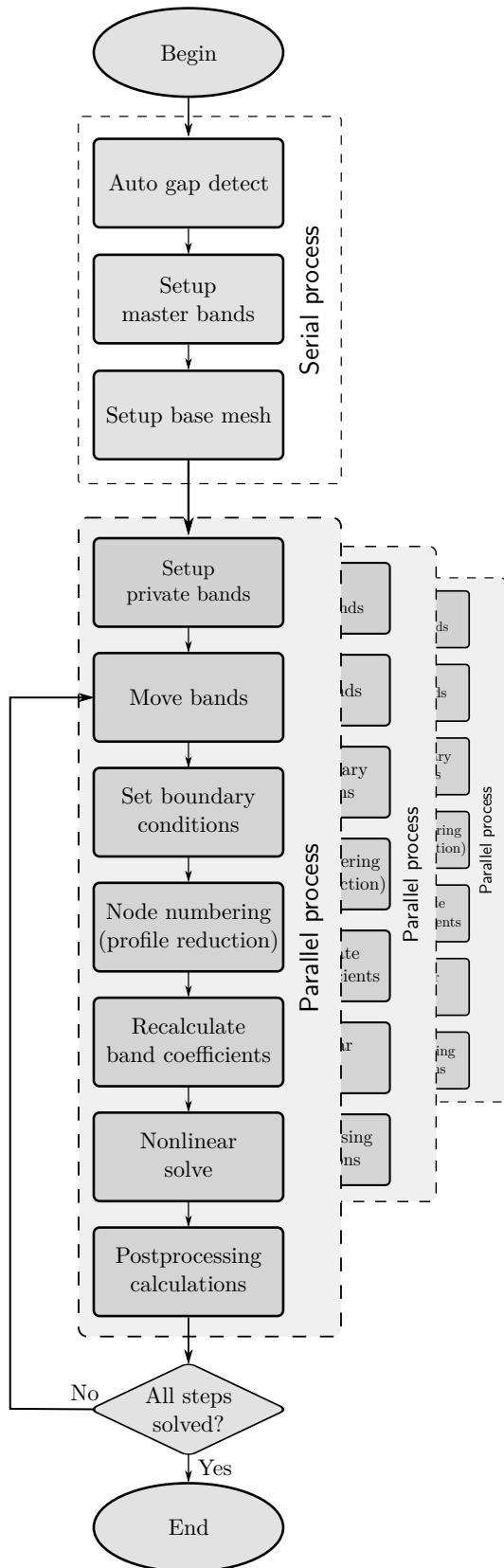
Figure 2: Flowchart of the moving band solver.

Once the bands have been correctly setup for a specific time instant, the information is appended to the base mesh to form a mesh specific to this time instant, complete with elements in the air-gap regions. The next step is to apply the appropriate boundary conditions to this mesh.

Because of the difference in the number of nodes and their numbering, a new mapping of nodes to unknowns is required for each time-step and similarly, profile reduction is executed for each time-step. It is noted that with the air-gap element technique previously implemented in *SEMFEM*, profile reduction was only performed once before proceeding to solving. However, the reduction in solution time due to the improved conditioning of the system matrix obtained with the moving band technique far outweighs the cost of the additional profile reductions, especially for multiple air-gaps with many nodes.

Prior to solving, the matrix coefficients related to the elements in the air-gap must be recalculated. The coefficients for the rest of the model are constant and are calculated only once, before starting the time-stepping procedure. The nonlinear problem is solved using the Newton-Raphson method.

Finally, the post-processing calculations of the torque, flux-linkage and copper loss are also performed in parallel.

4. VERIFICATION AND PERFORMANCE EVALUATION

In order to evaluate the accuracy and performance of the moving band solver, this section reports simulation results for two machines. The first machine is an interior permanent magnet (IPM) machine, shown in Figure 3. The second is the magnetically geared machine introduced in Figure 1. Both machines were simulated using air-gap elements as well as moving bands with a coarser and a finer mesh. The only difference between the air-gap element and the moving band models lies in the modeling of the air-gap regions. The meshes in the rest of the models were exactly the same. However, the underlying torque calculation method for the two movement methods also differ. For the air-gap element, the maxwell stress tensor method, as described in [3] is used. For the moving band method, the Coulomb virtual work method [8, 9] is used.

4.1 Interior permanent magnet machine

A time-stepped simulation consisting of 200 static solutions was performed for the IPM machine. Figure 4 shows a comparison of the torque calculated using the two movement handling methods for a relatively coarse mesh. It can be seen that the average torques are in good agreement, however, the torque calculated using the MB has a high frequency oscillation. This illustrates the superior accuracy of the AGE in coarse meshes. Typically when using the MB, the air-gap region would be meshed
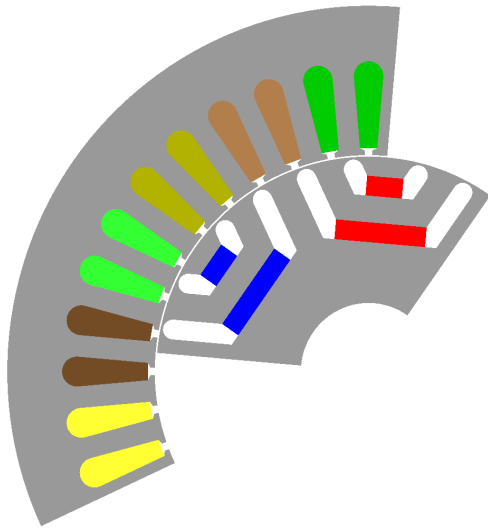
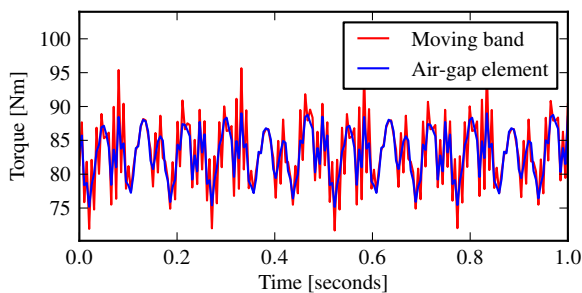Figure 3: Single air-gap, periodic model of a synchronous PM reluctance machine.

Figure 4: IPM machine: Torque comparison with a coarse mesh.
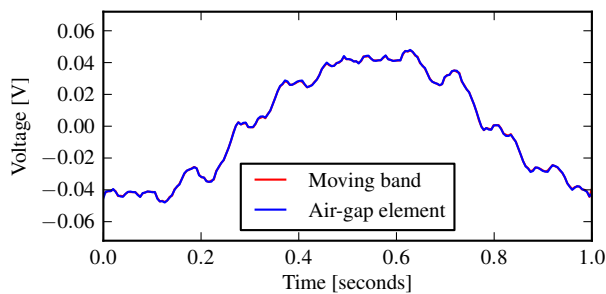

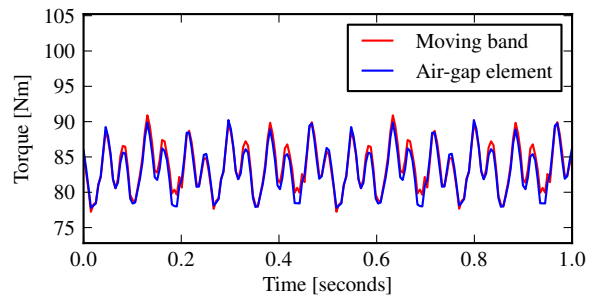Figure 5: IPM machine: Voltage comparison with a coarse mesh.

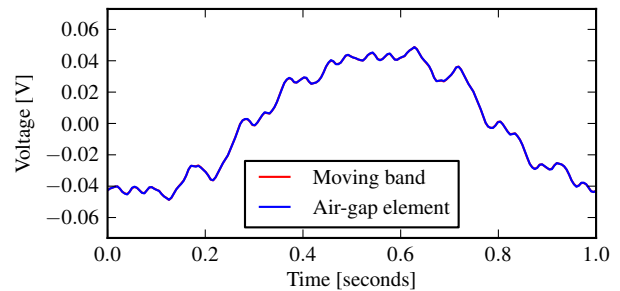Figure 6: IPM machine: Torque comparison with a fine mesh.

Figure 7: IPM machine: Voltage comparison with a fine mesh.

finer than when using the AGE in order to improve the accuracy, with some added cost in computational time. However, such refinements are not considered in this paper. Figure 5 shows a comparison of the phase voltage for the coarse mesh. The results are in very good agreement.

The results for a simulation with a finer mesh are shown in Figures 6 and 7. Note that the calculated torques match very closely in this case. Once again, there is almost no difference in the calculated phase voltage.

4.2 Magnetically geared machine

The simulation of the MGM consisted of 50 time steps. Figure 8 shows a comparison of the calculated torques in each of the three air-gaps of the machine using the two movement methods. These results are for a relatively coarse mesh. Clearly, the results are in good agreement, although the moving band method's results for the outer air-gap does have a small oscillation. The phase voltages, shown in Figure 9, match very closely. Similar results have been achieved with a finer mesh, the only difference being a reduction in the torque ripple calculated using the MB.

4.3 Performance comparison

The simulations were run on an Intel i7 CPU with 4 cores (8 virtual cores). Both methods exploit the multi-core architecture to run calculations in parallel. The calculation times for the different simulations are given in Table 1. Note that in all cases the MB was significantly faster
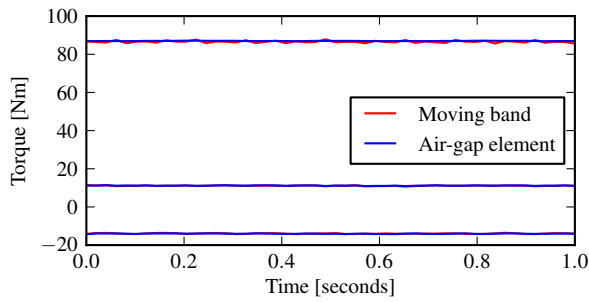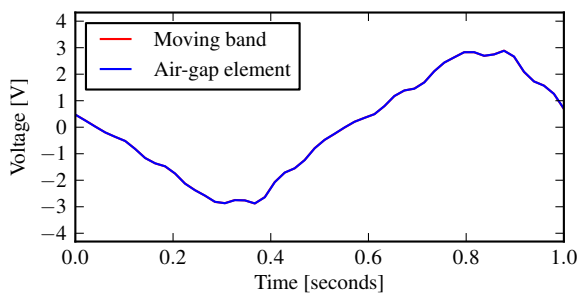
Figure 8: Torque comparison.

(a) Air-gap element   (b) Moving band
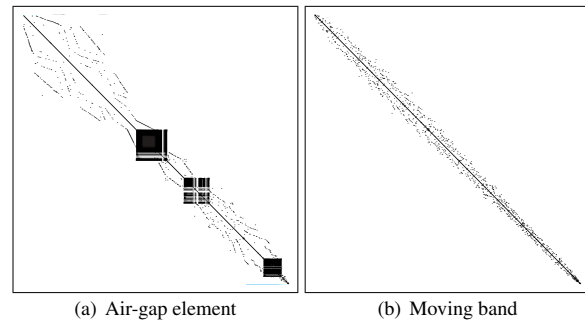
Figure 10: Matrices generated by FEM.

Figure 9: Voltage comparison.

5. CONCLUSIONS

In this paper, the most prominent movement facilitating methods used in finite element analysis of electrical machines were briefly reviewed. The simulation of magnetically geared machines is very demanding when movement is considered. The difficulty is due to multiple air-gaps and lack of periodicity in the models. Considering these properties, the moving band method is a good choice when a direct method of solving the final system equation is employed. This is because it produces a highly sparse system matrix with the lowest bandwidth of all the methods considered. The method has been implemented in an in-house finite element package and good agreement with results obtained with air-gap elements have been demonstrated. For an example magnetically geared machine, the computational time when using the moving band is roughly 20 times less than when using the air-gap element. On the other hand, the superior accuracy of the air-gap element when using relatively coarse meshes have been demonstrated and it remains a valuable method, especially when the model size can be reduced by exploiting periodicity.

than the AGE, but for the MGM a vast improvement in computational times is observed. The MB is roughly 20 times faster than the AGE for the MGM. Considering that the accuracy of the torque calculation for the MGM was also acceptable, the MB is definitely the preferred choice for the simulation of MGMs.

In order to explain why the MB band performs so much better than the AGE in the case of the MGM, Figure 10 illustrates the structure of the final system matrices obtained using the AGE and the MB. The contribution of the three air-gap elements are clearly present in the AGE matrix in the form of the three dense blocks. The profile of the AGE matrix is also higher than that of the MB matrix. The MB matrix can be solved efficiently using the direct method employed in *SEMFEM*.

6. FUTURE WORK

Among the many techniques of solving the final system equation, the ICCG method [10] is a widely used method. The method has the advantage that it is not strongly affected by the bandwidth of the system matrix [11]. For this reason, the other methods considered in this paper may show a significant improvement in performance if this type of equation solver is used. These possibilities should be further investigated.

The problem of noisy results due to the distortion of the moving band elements can be avoided by using higher-order elements in the moving band, as demonstrated in [12]. If the accuracy of the first order implementation proves to be insufficient in some cases, it is recommended that higher-order hierarchic elements be used in the moving band.

Simulation case	Calculation times [seconds]	
	AGE	MB
IPM machine (2355 nodes)	19.7	9.4
IPM machine (7076 nodes)	108.5	68.9
MGM (9858 nodes)	971.8	37.4
MGM (28436 nodes)	7074	376.4

Table 1: Performance comparison of movement methods

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