Effect of Stator Chording and Rotor Skewing on Performance of Reluctance Synchronous Machine

Xola B. Bomela, *Member, IEEE*, and Maarten J. Kamper, *Member, IEEE*

***Abstract*—Low torque ripple in electrical machines is generally required to reduce acoustic noise and mechanical resonance vibration. To design for low torque ripple, however, affects the average torque and the power rating of the machine. In this paper, the effect of stator winding chording and rotor skewing on the average torque, power factor, and torque ripple of the normal laminated, internal flux barrier rotor reluctance synchronous machine is investigated. The two-dimensional finite-element time-step method together with the basic machine equations are used in the analysis. It is shown that to design, in general, for low torque ripple and minimal effect on torque rating of the reluctance synchronous machine, full-pitch stator windings must be used, the rotor must be skewed by a stator slot pitch, and a low number of stator slots must be avoided.**

***Index Terms*—Chording, power factor, reluctance synchronous machine, skewing, torque, torque ripple.**

NOMENCLATURE

b_p	Flux barrier pitch at rotor surface (m).
C, k	General machine constant.
c	Coil span in terms of number of slots per pole.
d_i	Air-gap diameter (m).
g_d, g_q	Resultant air-gap lengths of the d- and q-axes magnetic circuits of the unskewed machine (m).
I_s	RMS phase current (A).
k_d, k_q	Coefficients considering the magnetomotive force (MMF) drop across the iron of the d- and q-axes magnetic circuits.
s_d, s_q	Skew factors of the d- and q-axes magnetic circuits.
k_{dn}	Distribution factor of nth MMF space harmonic.
k_{cn}	chording factor of the nth MMF space harmonic.
k_{wn}	winding factor of the nth MMF space harmonic.
L_{dm}, L_{qm}	d- and q-axes magnetizing inductances (H).
ℓ	Core length (m).
m	Number of phases.
N	Number of stator slots.
F_n	Amplitude of the nth-order MMF harmonic (At).
p	Number of pole pairs.
q	Number of slots per pole per phase.
W	Number of turns in series per phase
ϕ	Angle between the current space phasor and the d axis (rad).
τ_n	Pole pitch of the nth MMF space harmonic (m).
τ_s	Stator slot pitch (m).

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X. B. Bomela was with the Department of Electrical and Electronic Engineering, University of Stellenbosch, 7600 Stellenbosch, South Africa. He is now with LIW, 0140 Lyttelton, Pretoria, South Africa (e-mail: xolab@liw.denel.co.za).

M. J. Kamper is with the Department of Electrical and Electronic Engineering, University of Stellenbosch, 7600 Stellenbosch, South Africa (e-mail: kamper@ing.sun.ac.za).

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I. INTRODUCTION

RELUCTANCE synchronous machines (RSMs) are increasingly becoming a subject of research as they could be considered as the alternative to their other counterparts, namely, permanent-magnet, induction, and switched reluctance machines. The RSM is a type of synchronous machine which makes use of a standard three-phase stator and an axially or normal (transverse) laminated rotor. The normal laminated type of rotor can either be a salient-pole rotor or a round rotor with a relatively high number of internal air openings called flux barriers. As the RSM is a standard three-phase ac machine, two common design aspects of the machine are important. These are the chording of the stator winding and the skewing of the rotor. It is common practice in ac machine design to minimize the torque ripple of the machine by chording the stator winding and by skewing the rotor. The stator winding is chorded to reduce the lower order magnetomotive force (MMF) space harmonics, while the rotor is skewed conventionally by a stator slot pitch to reduce the so-called slot harmonic torques. As axially laminated types of rotors or normal laminated internal flux barrier types of rotors of the RSM are quite different from other types of rotors used in ac machines, such as the salient-pole, cage, or permanent-magnet rotors, the question arises: "What effect do stator chording and rotor skewing have, in general, on the performance of the RSM, and is chording, for example, necessary?"

Among the most important publications on torque-ripple evaluation of the RSM are [1]–[4]. In [1] and [2], extensive mathematical derivations have been used in studying the torque ripple of the RSM with internal flux barrier types of rotors. The theoretical approach followed in [2] is confirmed by finite-element analysis results presented in [4]. In [1] and [3], it is explained that skewing the rotor of the RSM does reduce torque ripple; however, a remaining torque ripple component can still be present. In [2], the aim was to design

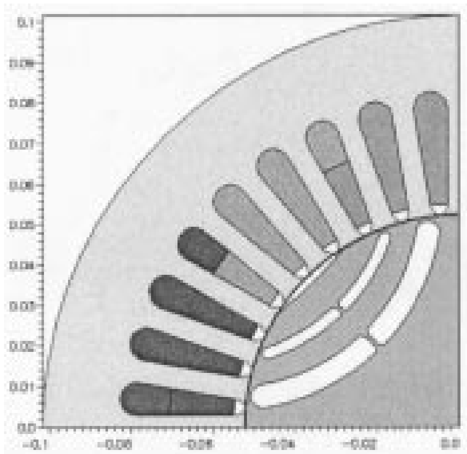

Fig. 1. One pole segment of 9-kW RSM [7].

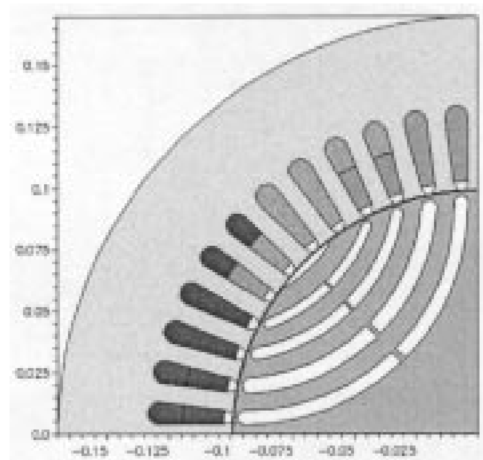

Fig. 2. One pole segment of 45-kW RSM [8].

the rotor to obtain as low as possible torque ripple for servo applications. In this design, a sinusoidal MMF was assumed, i.e., a chorded or distributed stator winding with negligible MMF space harmonics was assumed. All the rotors used in the measurements were skewed by one stator slot pitch. Some experimental results were given by [2] showing the effect of chorded windings on the torque ripple. In [5] and [6], it is shown that skewing the rotor reduces the torque ripple of the RSM significantly. However, a significant 10% reduction in rated power output of the machine was found. All these results will be referred to in this paper.

No attention was given in the literature to the effect of chorded or unchorded stator windings on the torque ripple and the power rating of the RSM. Also, the effect of skewing on the power rating of the RSM has not been investigated thoroughly. This paper addresses these aspects. The aim is to give general directives with regard to chording and skewing in the design of the RSM. The focus of the paper is not specifically on the design of the RSM rotor to minimize torque ripple, as in [1]–[3], but to show, in general, the effects of chording and skewing. Only normal laminated, internal flux barrier rotor RSMs are considered.

The method of investigation used in the paper is rather simple, as the basic machine equations together with finite-element analysis are used. The two–dimensional (2-D) finite-element time-step method is used in the analysis. The finite-element analysis method takes into account the effects of saturation, cross magnetization, and skewing [7]. The effect of stator winding chording on the torque ripple, average torque, and power factor of the RSM is investigated by varying the fractional pitch windings. The effect of rotor skewing is investigated by varying the skew in terms of stator slot pitches.

Two RSMs are considered in the finite-element analysis as well as in laboratory measurements. These are a small 36-slot stator machine and a medium-power 48-slot stator machine. Figs. 1 and 2 show the cross sections of the stator and the rotor structures of these two four-pole, optimum designed, normal laminated rotor RSMs. The design details and machine dimensions are given in Table I. Both machines were built and tested under current vector control. The results are published in [7] and [8].

TABLE I
DESIGN DETAILS OF RSMs

	9 kW RSM	45 kW RSM
Stator outer diameter (mm)	203	340
Stator inner diameter (mm)	105	199
Stack length (mm)	133	175
Stack volume (dm^3)	4.3	15.9
Number of poles	4	4
Stator chording	Varies	varies
Number of stator slots	36	48
Skew (slot pitches)	Varies	varies
Airgap length (mm)	0.34	0.62

II. EFFECT OF STATOR WINDING CHORDING

The equation for the amplitude of the nth-order rotating MMF wave in an ac machine with a three-phase winding is given by

$$F_n = \frac{1.35}{p} W I_s \left(\frac{k_{wn}}{n}\right). \qquad (1)$$

There is a fundamental, $n = 1$, and harmonics of orders 5, 7, 11, 13, or $6a \pm 1$, where a is any positive integer. In the case of the RSM the p pole-pair fundamental MMF wave $(n = 1)$ moves synchronously with the p pole-pair rotor. The harmonic MMF waves with np pole pairs, however, move asynchronously with the p pole-pair rotor at speeds $1/n$ that of the rotor. These asynchronously MMF harmonic waves, although n times smaller in amplitude than those of the fundamental when $k_{wn} = 1$ in (1), may cause torque ripple in the machine. The harmonic torque caused by the nth-order MMF and nth-order flux linkage harmonics may be expressed as

$$T_n = kF_n\lambda_n \sin(\gamma_n) \qquad (2)$$

where λ_n is the amplitude of the nth-order flux linkage harmonic. γ_n is the electrical phase angle between the nth-order MMF and nth-order flux linkage harmonics. This angle is not a constant but varies with rotor position.

To reduce the harmonic torque, the amplitudes of the most important MMF harmonics, i.e., the lower order fifth and seventh harmonic amplitudes, F_5 and F_7, must be reduced in (2). This can be done effectively by reducing the winding factors k_{w5} and k_{w7} in (1) by chording the stator winding. The effect of this chording on the general performance of the RSM is studied in the following sections.

A. Effect of Chording on Average Torque and Power Factor

To investigate the effect of chording on the average (fundamental) torque and power rating of the RSM, the torque equation of the machine is considered. This equation, as derived in all classical texts on dq-axes theory, is given by

$$T = \frac{3}{2}p(L_{dm} - L_{qm})I_s^2 \sin(2\phi). \tag{3}$$

In [9], the equations for both the d- and q-axes magnetizing inductances are given as follows:

$$L_{dm} = \frac{m(Wk_{w1})^2 d_i \ell \mu_0}{\pi p^2 (s_d g_d) k_d} \tag{4}$$

$$L_{qm} = \frac{m(Wk_{w1})^2 d_i \ell \mu_0}{\pi p^2 (s_q g_q) k_q}. \tag{5}$$

The effective d- and q-axes air-gap lengths are defined as

$$g_{de} = (s_d g_d) k_d \quad \text{and} \quad g_{qe} = (s_q g_q) k_q. \tag{6}$$

In (6), $k_d \geq 1$ and $k_q \geq 1$. The skew factors s_d and s_q are unity for an unskewed machine. For a skewed machine, $s_d > 1$ and $s_q < 1$, because with skew the resultant air-gap length increases in the d-axis direction due to less iron and decreases in the q-axis direction due to more iron. The magnetizing inductance difference using (4), (5), and (6) is

$$\Delta L_m = L_{dm} - L_{qm} = C_1 k_{w1}^2 \left(\frac{1}{g_{de}} - \frac{1}{g_{qe}} \right). \tag{7}$$

Hence, with constant rms current and current angle, (3) becomes

$$T = C_2 k_{w1}^2 \left(\frac{1}{g_{de}} - \frac{1}{g_{qe}} \right) \tag{8}$$

where $C_2 = (3/2)pC_1 I_s^2 \sin(2\phi)$.

Equation (8) shows that the torque of the RSM is a function of the square of the fundamental winding factor. For the nth MMF harmonic, the winding factor is given by

$$k_{wn} = k_{dn} \cdot k_{cn} = \frac{\sin(n\pi/2m))}{q \sin(n\pi/2mq)} \cdot \cos\left(\frac{n\pi}{2}[1 - c/mq] \right). \tag{9}$$

The fundamental, fifth, and seventh harmonic winding factors and the change in per-unit average torque using (8) and (9), assuming the last term of (8) as constant, are calculated for four-pole machines with $N = 24, 36, 48$ and 60 stator slots. The results are given in Table II. Hence, the 5/6, 7/9, 10/12, and 12/15 chorded windings should be used, since the main focus is to reduce the fifth and the seventh MMF harmonics. Using these conventionally chorded stator windings reduces the torque and, thus, the power rating of the RSM by 7%, 12%, 7%, and 10%, respectively (see Table II).

TABLE II
CALCULATED WINDING FACTORS OF FOUR-POLE MACHINES AND EFFECT ON PER-UNIT TORQUE.

24 slots: k_{wn}				
N	6/6	5/6	4/6	3/6
1	0.9659	0.933	0.8365	0.683
5	0.2588	0.067	-0.2241	-0.183
7	-0.2588	0.067	0.2241	-0.183
T(p.u.)	1.0	0.933	0.75	0.5
36 slots: k_{wn}				
N	9/9	8/9	7/9	6/9
1	0.9598	0.95	0.902	0.831
5	0.2176	0.1399	-0.0378	-0.1884
7	-0.1774	-0.0607	0.1359	0.1536
T(p.u.)	1.0	0.979	0.880	0.749
48 slots: k_{wn}				
N	12/12	11/12	10/12	9/12
1	0.9577	0.9495	0.925	0.8848
5	0.2053	0.1629	0.0531	-0.0786
7	-0.1576	-0.0959	0.0408	0.1456
T(p.u.)	1.0	0.983	0.933	0.854
60 slots: k_{wn}				
N	15/15	14/15	13/15	12/15
1	0.9567	0.9514	0.9358	0.9099
5	0.2	0.1732	0.1	0.0
7	-0.1494	-0.1111	-0.0156	0.0878
T(p.u.)	1.0	0.9891	0.9568	0.9045

According to the results of Table II, thus, chording has a significant effect on the torque rating of the RSM. Two other observations from the results of Table II are: 1) the per-unit torque of the lower number of stator slot machines is very sensitive to the amount of chording and 2) the fundamental winding factor k_{w1} and, thus, the fundamental MMF amplitude F_1 for full-pitch windings is almost not affected at all by the number of stator slots.

The average torque as a function of winding factor is also determined from finite-element analysis for the two RSMs of Figs. 1 and 2. In this analysis, the rotors are skewed by one stator slot pitch. The current angle of (3) is kept constant in the analysis at a typical optimum angle of $\phi = 65°$ [8], [10]. The results of the nonlinear finite-element solutions are shown in Figs. 3 and 4. Also shown are the per-unit torque calculations of Table II for the 36- and 48-slot RSMs. The nonlinear finite-element calculations, which take into account saturation and cross magnetization, show less drop in torque than the calculations of Table II. This is due to the last term of (8), which improves with chording as the machine comes out of saturation due to less flux and g_{de} decreases. The nonlinear finite-element results, however, still show a drop in torque and, thus, torque rating, of 8% and 4% at conventional 7/9 and 10/12 chordings, respectively.

In Figs. 5 and 6, the finite-element calculated torque versus current angle at per-unit current is shown for the two RSMs of Figs. 1 and 2, with chording as a parameter. The drop in torque due to chording is clear from these figures. Also shown in Figs. 5 and 6 are the calculated power factors versus current angle of the two RSMs. These power factors are calculated using the complete d–q equivalent circuits of the RSM [7]. The results show that chording has practically no effect on the power factor over

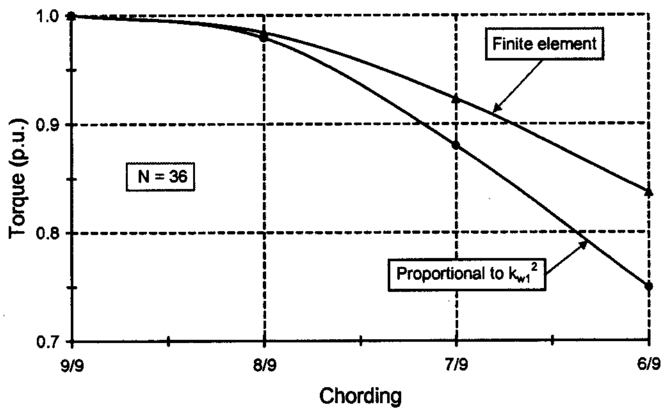

Fig. 3. Effect of chording on torque of small RSM.

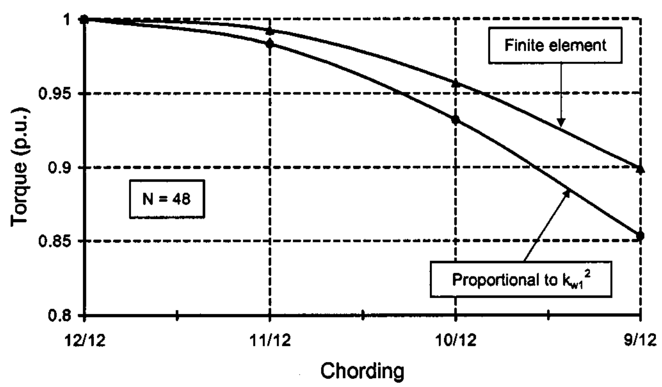

Fig. 4. Effect of chording on torque of medium-power RSM.

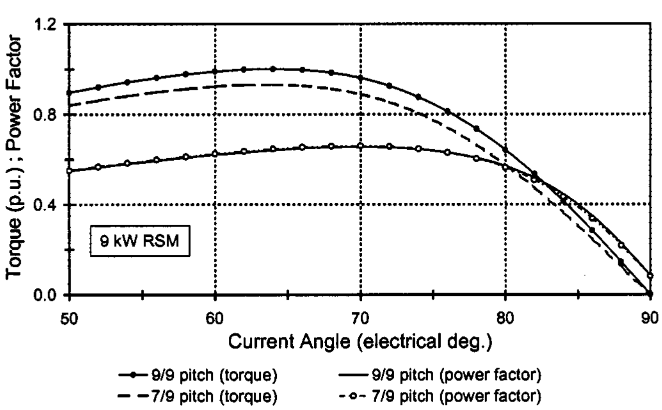

Fig. 5. Effect of chording on torque and power factor at per-unit current (small 9-kW RSM).

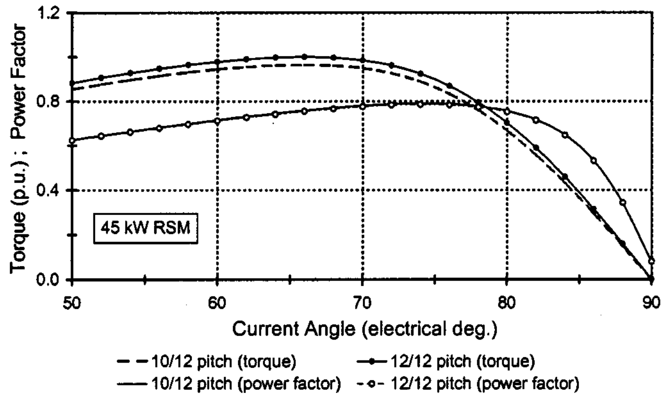

Fig. 6. Effect of chording on torque and power factor at per-unit current (medium-power 45-kW RSM).

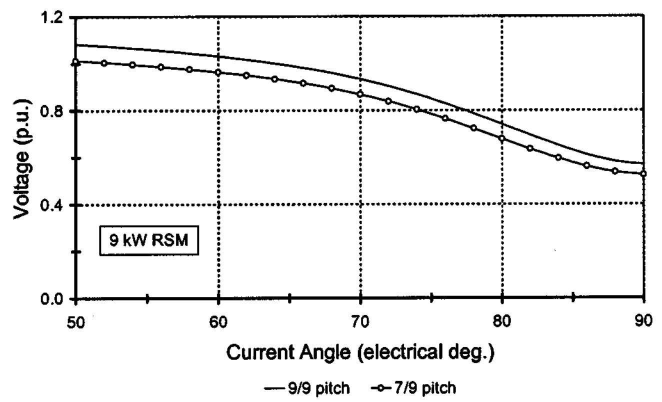

Fig. 7. Effect of chording on supply voltage of small RSM.

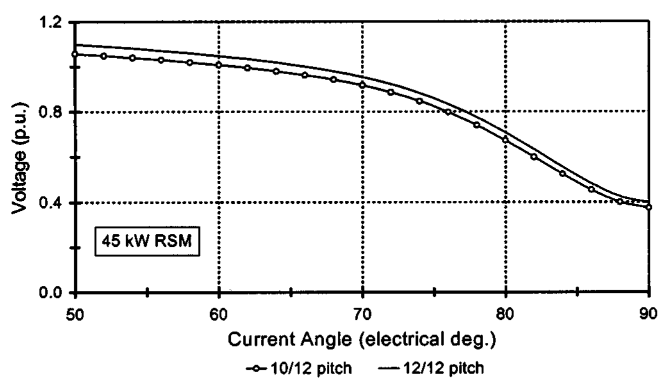

Fig. 8. Effect of chording on supply voltage of medium-power RSM.

the whole current angle range. The latter can be explained by the well-known fact that the power factor of the RSM is dependent on the inductance ratio $\sigma = L_{dm}/L_{qm}$ (ignoring the leakage inductance). From (4), (5), and (6), this ratio is given by

$$\sigma = \frac{L_{dm}}{L_{qm}} = \frac{(s_q g_q) k_q}{(s_d g_d) k_d} = \frac{g_{qe}}{g_{de}} \tag{10}$$

which shows that the power factor is theoretically not affected by stator winding chording.

Finally, Figs. 7 and 8 show the calculated supply voltages of the two RSMs versus current angle at per-unit current. The supply voltages of the full-pitch RSMs are higher. This is important for the field-weakening region of the machine. It can be seen from Figs. 5–8 that at large current angles (field-weakening region) and with the same voltage supply, the chorded RSM has a slightly higher torque and better power factor. This indicates that the performance of the chorded RSM in the high field-weakening region (large current angles) is slightly better than that of the full-pitch RSM.

B. Effect of Stator Chording on Torque Ripple

The effect of stator chording on the overall torque ripple of the RSM is investigated by means of finite-element time-stepping analysis on the two machines of Figs. 1 and 2. The results of this

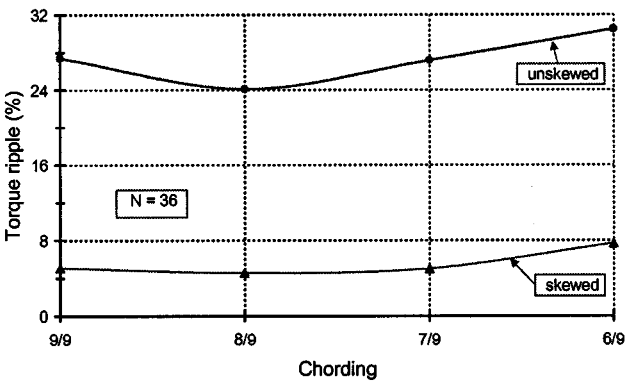

Fig. 9. Effect of chording on torque ripple of small RSM.

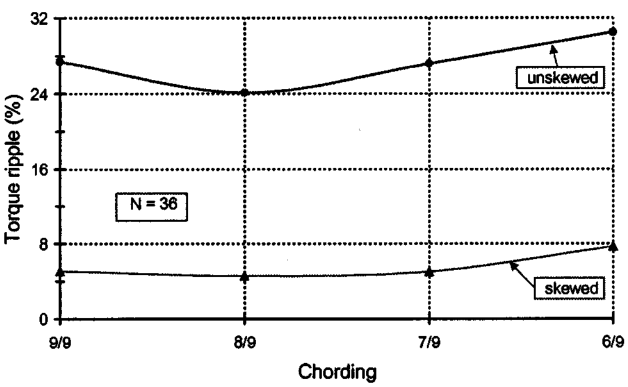

Fig. 10. Effect of chording on torque ripple of medium-power RSM.

study are shown in Figs. 9 and 10 for skewed and unskewed machines. Two interesting observations from the results are the following. Firstly, the optimum chording for reducing torque ripple does not necessarily coincide with conventional chording (7/9 and 10/12 in this case) to reduce the fifth and seventh MMF space harmonics. Secondly, chording appears to have, in general, very little effect on the torque ripple of the RSMs under consideration. In the case of the small RSM (Fig. 9) conventional chording shows no reduction effect at all on the torque ripple. The drop in torque ripple due to chording in the skewed cases is less than 1,5%, while in the unskewed cases it is 3.3% and 1.8% for the 36- and 48-slot machines, respectively. Finally, the measured results of [2] also indicate that the effect of the lower order MMF rotating harmonics on torque ripple, in general, is minimal.

The above observation that chording has almost no effect on the torque ripple of the RSM is very important and needs an explanation. Consider the cross sections of three two-pole RSMs shown in Figs. 11–13. In these figures, a fifth MMF rotating harmonic, as an example, is shown. This harmonic has ten poles and rotates asynchronously in the opposite direction of the two-pole rotor. Clearly, with the RSM rotor of Fig. 11 using only external cutouts to obtain saliency, torque pulsations are expected as the ten-pole harmonic field reacts with the two-pole rotor at a frequency of $12f$. With the uniformly distributed internal flux barrier round rotor of Fig. 12, however, the situation is different. It is clear from Fig. 12 that no fifth harmonic flux will flow

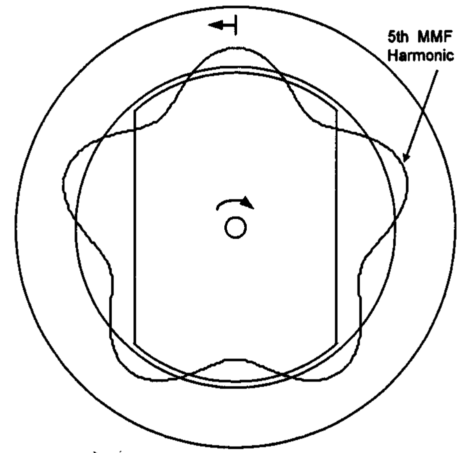

Fig. 11. RSM with salient pole rotor and MMF harmonic.

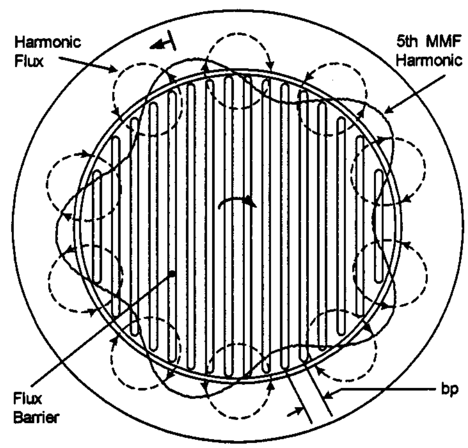

Fig. 12. RSM with round internal flux barrier rotor (MMF harmonic in q-axis position).

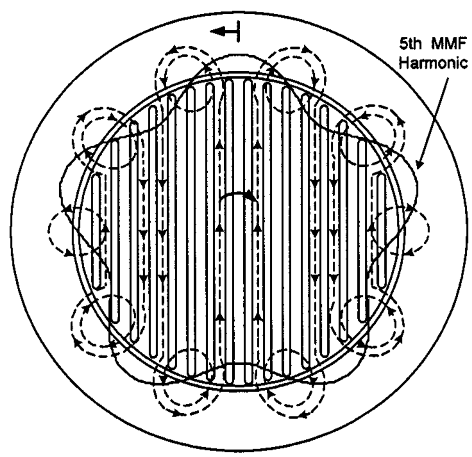

Fig. 13. RSM with round internal flux barrier rotor (MMF harmonic in d-axis positon).

through the rotor in the d-axis direction, but will only flow, as indicated, in the q-axis direction of the machine. This is an important observation. It implies that the lower order harmonic-generated fluxes are attenuated by the high q-axis reluctance magnetic paths through the flux barrier openings. Note that the netto q-axis harmonic flux over a pole pitch is zero. Furthermore, the

q-axis harmonic fluxes will help to saturate the iron bridges at the surface of the rotor, helping to reduce the main q-axis inductance. The latter will be beneficial to the torque development of the machine.

With the same rotor as in Fig. 12, but in a different position with respect to the fifth MMF harmonic, as shown in Fig. 13, the situation is different again. Here, the possibility is now that harmonic flux can also flow through the rotor in the d-axis direction to the other side of the machine. This might be seen as a low-reluctance path for the harmonic flux. However, this flux goes through relatively long and highly saturated iron regions of the rotor under full-load conditions. The reluctance of this magnetic path, thus, may well be considered as rather high than low. Note also in this case that the netto d-axis harmonic flux is zero.

Obviously, the above explanation is very simplistic and the combined effect of the lower order harmonics must rather be considered. Nevertheless, the explanation gives the indication that the lower order harmonic fluxes are attenuated by the internal flux barrier rotor of the RSM. The harmonic torques thus caused by the lower order harmonic MMFs and fluxes will be low and chording will have little effect in the further weakening of these harmonic torques.

Following the explanation above, it is clear that to reduce the harmonic flux caused by an MMF harmonic, the barrier pitch b_p must be at least less than the pole pitch of that MMF harmonic, or else, $b_p < \tau_n$ (see Fig. 12). A relatively high number of internal flux barriers, in general, thus, is required. It can be seen from Figs. 9 and 10 that the effect of chording on torque ripple is less for the medium-power RSM of Fig. 2 than for the small RSM of Fig. 1. The reason is simply that the former has a higher number of internal flux barriers on the rotor. The same is observed from the measured results of [2], where the RSM with a high number of internal flux barriers on the rotor shows almost no difference in torque ripple between using a chorded and a full-pitch winding.

Another general design requirement, however, is to keep the barrier pitch larger than the stator slot pitch τ_s. This is to avoid high flux pulsations in the rotor iron segments. As τ_s is inversely proportional to the number of stator slots, a high number of flux barriers on the rotor will require a high number of stator slots. In this regard, a higher number of stator slots is recommended to be used in RSMs. Table II shows that a higher number of stator slots, using full-pitch windings, has practically no effect on the fundamental winding factor k_{w1} and, thus, on the torque and power rating of the machine.

Finally, some finite-element-calculated results of the actual torque versus rotor position of the two RSMs under consideration are shown in Figs. 14 and 15. Hence, the effect of chording on the average torque and torque ripple of the machines, as discussed above, can be observed.

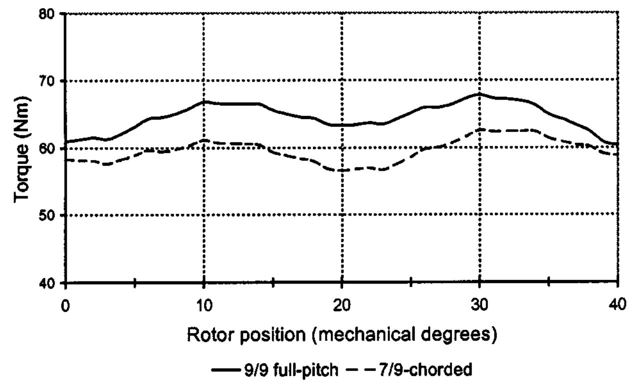

Fig. 14. Finite-element calculated torque versus rotor position of the small RSM of Fig. 1 ($\phi = 65°$, skew = one slot pitch).

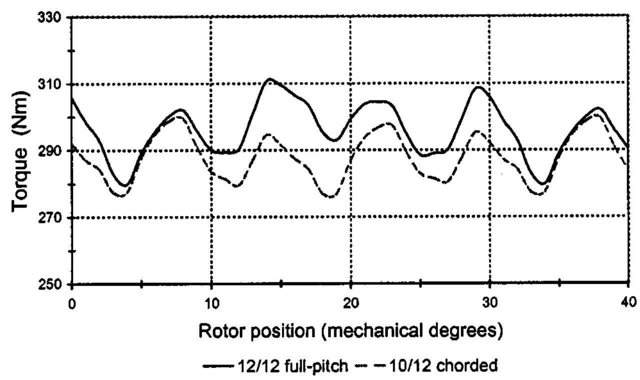

Fig. 15. Finite-element calculated torque versus rotor position of the medium-power RSM of Fig. 2 ($\phi = 65°$, skew = one slot pitch).

III. Effect of Skewing the Rotor

Equation (6) suggests that, when skewing the rotor, the skew factors s_d and s_q will increase and decrease the d- and q-axes air-gap lengths, respectively. Equations (7) and (10) then show that both ΔL_m and σ will decrease, and both the torque and power factor of the RSM will be negatively affected. This section investigates the effect of rotor skewing on the average torque, power factor, and torque ripple of the RSM.

A. Effect of Skewing on Torque and Power Factor

Skewing can be represented by using a set of unskewed machines of which the rotors are relatively displaced by an angle that is a fraction of the total skew. This is shown in Fig. 16, where the skewed machine has five submachines. Five submachines were found to give almost the same result as a continuous skewed machine. Figure 16(b) shows the representation of the unskewed machine with one submachine.

For each submachine, the rms current is the same, but the current angle ϕ and the inductance difference ΔL differ. Thus, from (3), the fundamental torque for submachine i for a given rms current is

$$T_i = C\Delta L_i \sin(2\phi_i) \tag{11}$$

where $\phi_i = \phi_1 + (i-1)\alpha$ and α is a skewed angle (Fig. 16), and ΔL_i is the inductance difference of submachine i. For the skewed machine, the average torque will be

$$T_{\text{skew}} = \frac{1}{k}\sum_{i=1}^{k} T_i \tag{12}$$

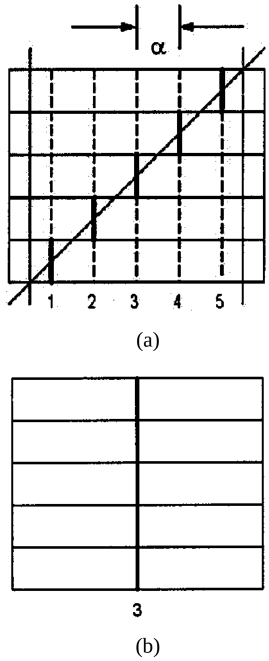

Fig. 16. (a) Representation of a skewed machine with five submachines. (b) Representation of an unskewed machine with one submachine.

where $k = 5$ from Fig. 16(a). For the unskewed machine, the average torque will be

$$T_{\text{unskew}} = T_3 = C\Delta L_3 \sin(2\phi_3). \quad (13)$$

The torque of the RSM as a function of ϕ has already been shown and explained by [10] and has typical curves like those shown in Fig. 17. It is important to understand that each submachine i has the same torque versus current angle curve. Operating the machine, thus, at a certain average current angle ϕ_3, it is clear from the torque–angle curves of Fig. 17 that the submachine torques T_i will be different and that $T_{\text{skew}} < T_3 = T_{\text{unskew}}$.

The question is: "How much will the skewed torque be less than the unskewed torque?" Consider in Fig. 17 the torque–angle curve at rated current ($I_s = 1.0$ p.u.). The different current angles and points on the torque curve are shown for five submachines. The average current angle ϕ_3 in this case is at an optimum value of 66°. It is clear that the difference between the skewed and unskewed torques, according to (12) and (13), will be small due to the flat torque–angle curve. However, with ϕ_3 at a higher current angle ($\phi_3 > 66°$) the difference might be significant.

To investigate the above in more detail, consider again the torque–angle curves of Figs. 17 and 18. It is clear from Fig. 17 that the shapes of the torque–angle curves at different loads are very much the same, except perhaps at low loads. Also from Fig. 18, it is clear that the torque–angle curves of different RSMs with different sizes and design are very much the same. The torque–angle curve of the 110-kW RSM in Fig. 18 is the curve of an RSM that uses a standard induction machine stator [11]. It is, thus, valid in investigating the general effect of skewing on the average torque of the RSM to use a typical torque–angle curve of a machine and apply that curve to RSMs with a different number of stator slots and poles.

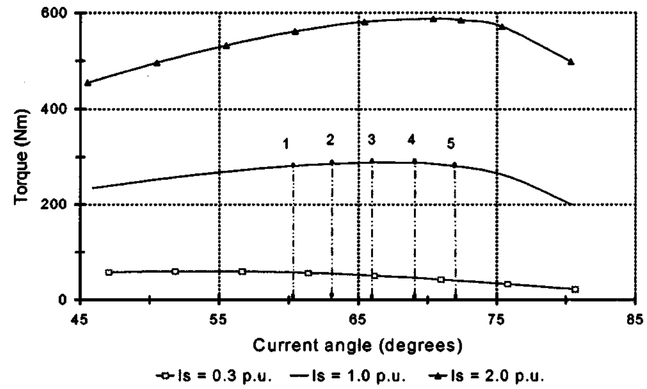

Fig. 17. Torque versus current angle of the RSM of Fig. 2 with current as a parameter (current angles are shown for five submachines with $\phi_3 = 66°$).

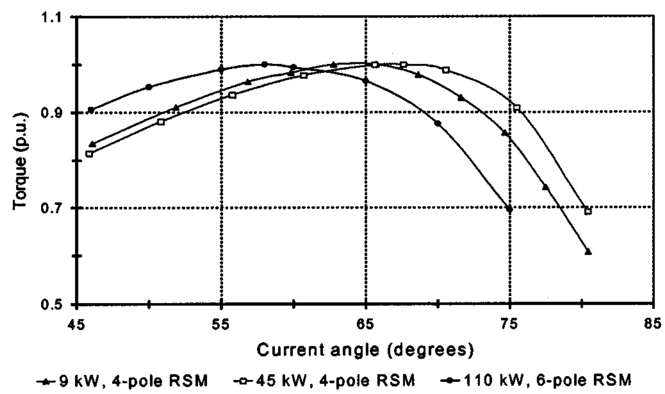

Fig. 18. Per-unit torque versus current angle of three RSMs.

Let us, therefore, obtain a per-unit torque equation of the rated torque–angle curve of Fig. 17 ($I_s = 1.0$ p.u.) and use this equation for two-, four- and six-pole RSMs with different numbers of stator slots and the rotors skewed by, conventionally, a stator slot pitch. A per-unit torque equation as a function of per-unit current angle can be obtained by applying a curve fitting, using, e.g., a fifth-degree interpolating polynomial of the Newton form on the rated torque–angle curve of Fig. 17. This gives the following equation:

$$T(\phi) = -71.344\phi^5 + 314.057\phi^4 - 550.823\phi^3 + 478.933\phi^2 - 204.944\phi + 35.121. \quad (14)$$

In (14), the maximum torque is 1.0 p.u. at an optimum current angle of 1.0 p.u. (1.0-p.u. current angle is 66° in this case). Using (14) in (12) and taking $k = 5$ submachines, the skewed torque can be calculated. The results of this analysis are shown in Table III. Hence, it is clear that the effect of skewing the rotor by one stator slot pitch on the torque, and, thus, the power rating of the RSM, is in general less than 2%. It depends on the number of stator slots. A low number of stator slots leads to more skewing of the rotor over a slot pitch and, consequently, a higher drop in torque. One may conclude from the results of Table III that skewing the rotor by one stator slot pitch has a negligible effect on the torque and the torque rating of the RSM, if for two-pole machines $N \geq 18$, for four-pole machines $N \geq 36$, and for six-pole machines $N \geq 54$.

TABLE III
PER-UNIT TORQUE AND CURRENT ANGLE OF RSMS WITH ROTORS SKEWED BY ONE STATOR SLOT PITCH

	T = 1.0 p.u. at ϕ = 1.0 p.u. for unskewed rotor					
	p = 1		P = 2		p = 3	
N	T (p.u.)	ϕ (p.u.)	T (p.u.)	ϕ (p.u.)	T (p.u.)	ϕ (p.u.)
12	0.952	0.945	≈ 0.8	≈ 0.85		
18	0.98	0.969			≈ 0.8	≈ 0.85
24	0.989	0.978	0.952	0.945		
36	0.995	0.985	0.98	0.969	0.952	0.945
48			0.989	0.978		
54					0.98	0.969
60			0.993	0.982		
72					0.989	0.978

Note from Table III that the optimum current angle for a skewed RSM is slightly less than that for an unskewed machine. The reason for this is clear from the torque–angle curves of Fig. 17, where the drop in torque for current angles larger than 1.0 p.u. is more rapid than for current angles less than 1.0 p.u. From this, it is also clear that operating the RSM at current angles larger than 1.0 p.u., that is, in the field-weakening region of the machine, the effect of skewing on the developed torque of the machine will be significant.

A final observation from the results of Tables II and III on four-pole RSMs is that it is better to use a 48-slot stator than a 36-slot stator. There is practically no difference in the values of the fundamental winding factors, which affects the torque according to (8), and the effect of skew is less. Furthermore, a higher number of flux barriers can be used due to the higher number of stator slots. This will reduce more effectively the lower order space harmonic fluxes according to the explanation in Section II.

To investigate the actual difference between the average torques of the skewed and unskewed RSMs, the finite-element calculation method is used for the two RSMs of Figs. 1 and 2. The results of these calculations are shown in Figs. 19 and 20. It is observed that skewing the machine by one stator slot pitch, which is commonly used, causes a drop in average torque of less than 2%. Skewing the machine by two stator slot pitches causes the average torque to drop by 11% for the small RSM and 5% for the medium-power RSM. From this analysis, thus, skewing the rotor by more or less one stator slot pitch is shown to have a negligible effect on average torque for 36- and 48-slot RSMs. This is in agreement with the results of Table III.

The result from above that skew has a minimal effect on the power rating of the RSM is in contrast to the results found by [5] and [6]. In both these publications, it is reported that skewing the rotor by a slot pitch reduces significantly the power rating of the RSM by 10%. This discrepancy can be explained as follows. In [6] a four-pole 24-slot stator RSM was used. According Table III, skewing a four-pole 24-slot RSM and operating the machine at an optimum current angle of 0.945 p.u. reduces the average torque of the machine by 4.8%. However, at a current angle of 1.0 p.u., the analysis shows a drop in torque of 6.3% and at 1.05-p.u. current angle (70°) the drop is 11%. This shows that operating the skewed machine not at an optimum current angle

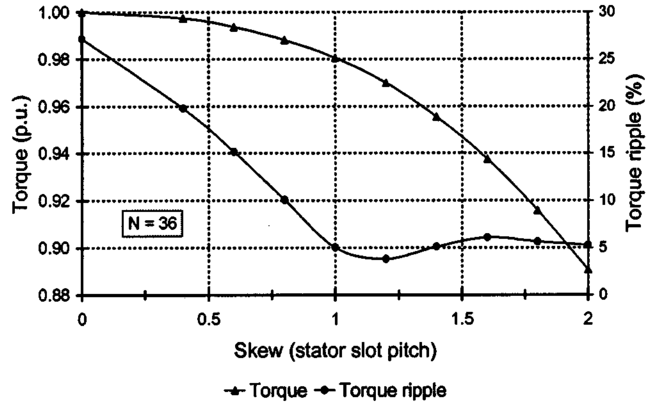

Fig. 19. Effect of skew on per-unit torque and torque ripple (36 stator slots, small RSM, 7/9 chorded).

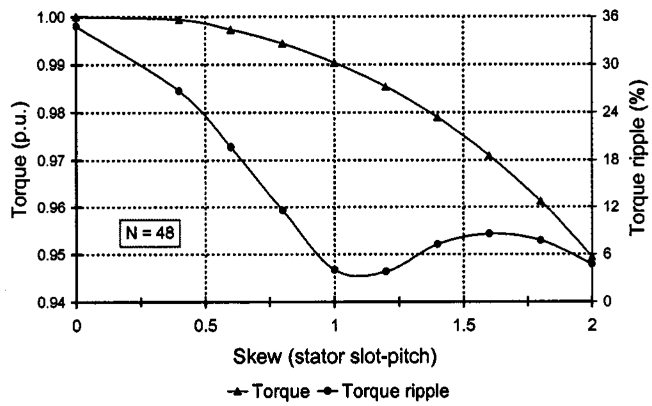

Fig. 20. Effect of skew on per-unit torque and torque ripple (48 stator slots, medium-power RSM, 10/12 chorded).

can have a significant effect on the torque of the machine. The latter might be the reason why the drop in rated torque found in [5] and [6] is so high. Furthermore, a low-number stator slot RSM was used by [6] so that the effect of skew on the torque of the machine is more. A final remark is that the RSMs tested in [5] and [6] were very small RSMs with rated torques less than 5 N·m. The torque–angle curves of these very small RSMs might differ from the curves given in Fig. 17 so that the effect of skew is more severe. The RSMs investigated in this paper are in the 50–1000-N·m torque range.

Finally, the performance in terms of torque, power factor, and supply voltage of the skewed and unskewed RSMs of Figs. 1 and 2 versus current angle at rated current are shown in Figs. 21–24. At rated current angles (typically, 65°), there is almost no difference in the torque and power factor between the skewed and unskewed RSMs. The power-factor curves at rated current angles are very linear, which explains, as for the torque, why skew has so little effect on the power factor in this current-angle region.

In the field-weakening region (large current angles), however, the effect of skew on the performance of the RSM is significant. With the supply voltages of Figs. 23 and 24 almost the same, the unskewed RSM outperforms completely the skewed machine in terms of torque and power factor in this current-angle region.

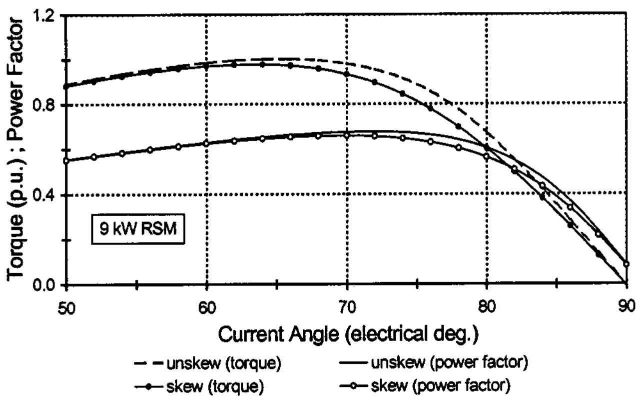

Fig. 21. Effect of skew on per-unit torque and power factor of the small 9-kW RSM at rated current.

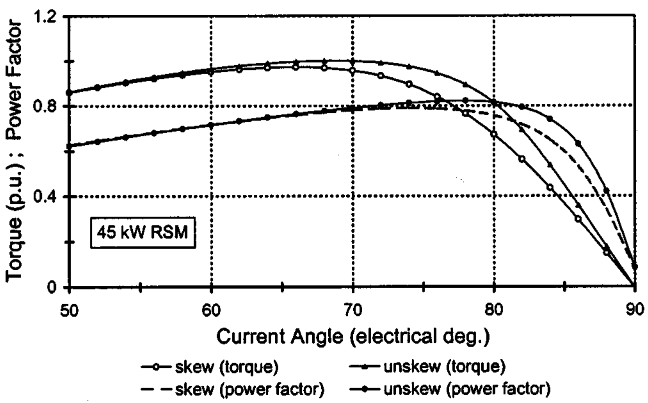

Fig. 22. Effect of skew on per-unit torque and power factor of the medium-power 45-kW RSM at rated current.

The effect of skew on the torque and power factor of the RSM can be explained in a different way by considering the effect of the air-gap lengths $g_{de} = (s_d\, g_d)k_d$ and $g_{qe} = (s_q\, g_q)k_q$ of (6) on ΔL_m and σ of (7) and (10), respectively. At less than rated current angles, the machine is deep into magnetic saturation due to the relatively high d-axis current. At this condition, it can be shown through magnetic circuit analysis that $(s_d g_d)k_d \approx C_3$ and $(s_q g_q)k_q \approx C_4$, which implies that g_{de} and g_{qe} are constants. Hence, the torque and power factor in this case will not be affected by skew, as is also clear from Figs. 21 and 22. However, in the field-weakening region (large current angles), the machine is out of magnetic saturation due to the relatively low d-axis current. It can then be shown through linear magnetic circuit analysis that $g_{de} = s_d g_d + C_5$ and $g_{qe} = s_q g_q + C_6$, which implies that the torque and power factor of the RSM in this case are directly affected by the skew factors s_d and s_q and, thus, by skew. The latter can be observed from Figs. 21 and 22.

B. Effect of Rotor Skew on Torque Ripple

The effect of skewing the RSM rotor on the torque ripple of the machine is more complex to study and needs finite-element calculations. This has been done for the two RSMs of Figs. 1 and 2. The percentage torque ripple is calculated as the maximum difference between the torque and the average torque, as a percentage of the average torque. The finite-element results are shown in Figs. 19 and 20. From this, it is clear that the torque

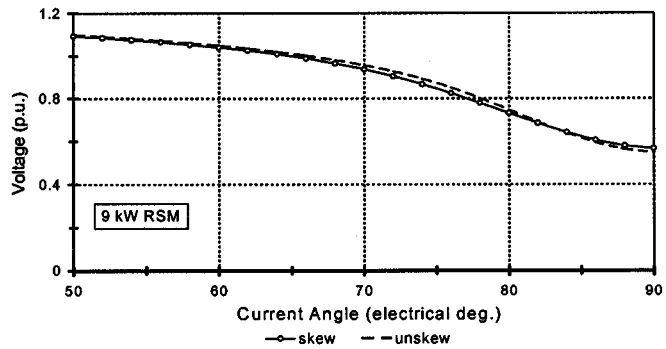

Fig. 23. Effect of skew on per-unit voltage of the small 9-kW RSM at rated current and rated speed.

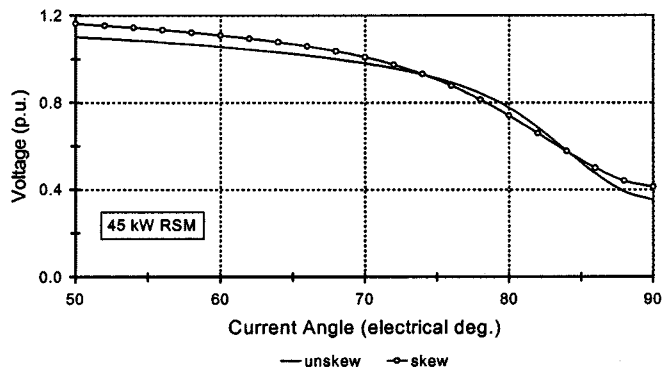

Fig. 24. Effect of skew on per-unit voltage of the medium-power 45-kW RSM at rated current and rated speed.

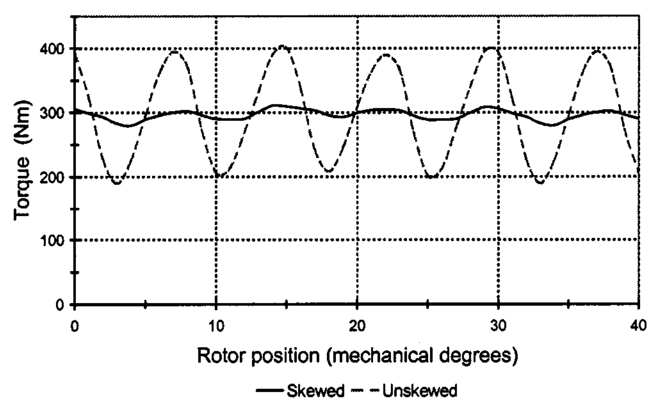

Fig. 25. Finite-element calculated torque versus rotor position of the medium-power RSM of Fig. 2.

ripples of both machines are drastically reduced from more than 30% to less than 5% by rotor skewing. A further observation is that the optimum skew is not necessarily at one stator slot pitch. For the two RSMs, the optimum skewings are at 1.2 and 1.1 stator slot pitches, respectively.

Finally, in Fig. 25, examples are shown of the torque ripples of the medium-power RSM (Fig. 2) using an unskewed rotor and a rotor that is skewed by a stator slot pitch. It clearly shows the effective reduction of torque ripple by means of rotor skewing.

IV. Some Calculated and Measured Results

In Fig. 26, the measured and calculated results of torque versus current angle are shown for the medium-power RSM.

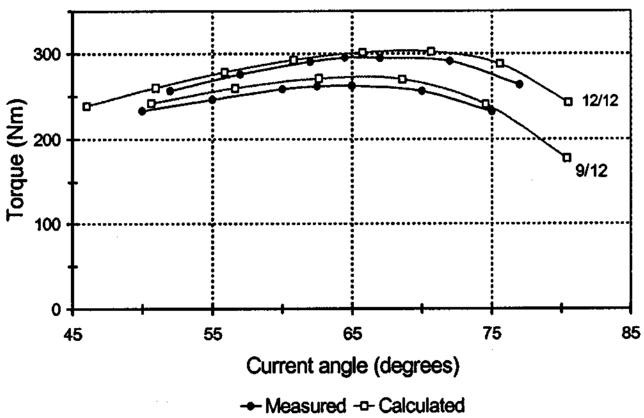

Fig. 26. Calculated and measured results of torque versus current angle with chording as a parameter for the medium-power RSM of Fig. 2.

The results are given for two different chorded windings with the machine at rated current. Clearly, there is a good agreement between calculated and measured results. The effect of chording on the output torque is in agreement with the discussions in Section II.

V. Conclusion

From the calculated results in this paper, the following conclusions are drawn with regard to the effect of stator winding chording and rotor skewing on the performance of the internal flux barrier rotor RSM.

1) Conventional chording of the RSM can easily have a 5%–10% reduction effect on the torque rating of the machine. The power factor is not affected. In the field-weakening region, chording has an advantage in terms of a slightly higher torque and better power factor for the same kilovoltampere supply.
2) Stator winding chording has little effect on the attenuation of the torque ripple of the RSM. The indication is that the lower order MMF harmonic-generated fluxes are attenuated by the internal flux barrier rotor. With regard to torque ripple, thus, chording is not necessary.
3) Skewing the rotor of the RSM by conventionally one stator slot pitch has little effect (less than 2%) on the torque, power factor, and power rating of the machine if the machine is operated at an optimum current angle. However, the latter is only true for RSMs with a certain minimum number of stator slots. In the field-weakening region, the unskewed RSM outperforms completely the skewed RSM in terms of torque and power factor for the same kilovoltampere supply.
4) Skewing the rotor by one stator slot pitch effectively reduces the torque ripple of the RSM.
5) A general design criterion for the RSM for low torque ripple and minimal effect on torque rating is to use full-pitch stator windings, to skew the rotor by a stator slot pitch, and to avoid a low number of stator slots. However, to have good performance in the field-weakening region, chorded unskewed machines must rather be used. The emphasis then, however, will be to design specifically for low torque ripple unskewed machines. Chording has the further well-known advantage of reduced endturn length and reduced overall axial length of the machine.

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Xola B. Bomela (S'00–M'00) was born in South Africa in 1968. He received the B.Sc. degree from the University of Fort Hare, Alice, South Africa, the B.Eng. degree from the University of Cape Town, Cape Town, South Africa, and the M.Eng. degree from the University of Stellenbosch, Stellenbosch, South Africa, in 1992, 1997, and 2000, respectively.

He is currently with LIW, Pretoria, South Africa. His main interests are electrical machine design and control and variable-speed drives.

Maarten J. Kamper (M'96) was born in South Africa in 1959. He received the B.Eng., M.Eng., and Ph.D. degrees from the University of Stellenbosch, Stellenbosch, South Africa, in 1983, 1987, and 1996, respectively.

In 1989, he joined the lecturing staff of the University of Stellenbosch, where he is currently an Associate Professor of electrical machines and drives. His main interest is electrical machine design using finite-element analysis and optimization algorithms. He is currently focusing on reluctance, axial flux permanent-magnet, and transverse flux machines for traction/wheel drive and generator applications.

Prof. Kamper is a Registered Professional Engineer in South Africa.