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PART II OF TWO PARTS

PAPERS

- Inclusive Model of Ferromagnetic Hysteresis. *B. F. Phelps, F. Liorzou, and D. L. Atherton* 1326
- Modeling Eddy Currents With Boundary Conditions by Using Coulomb's Law and the Method of Images.
. *K. Lee and K. Park* 1333
- Analytical Method for Eddy Current Loss in Laminated Rotors With Magnetic Bearings *Y. Sun and L. Yu* 1341
- Influence of Stator Slot Geometry and Rotor Eccentricity on Field Distribution in Cylindrical Magnetic Actuators.
. *L. S. Stephens and M. A. Casemore* 1348
- Two-Dimensional Cartesian Air-Gap Element (CAGE) for Dynamic Finite-Element Modeling of Electrical Machines With
a Flat Air Gap *R. Wang, H. Mohellebi, T. J. Flack, M. J. Kamper, J. D. Buys, and M. Feliachi* 1357
- Study of Triangular Winding Alternators *J. R. Quesada, V. Lemarquand, J.-F. Charpentier, and G. Lemarquand* 1361
- Obtaining the Operating Characteristics of Linear Induction Motors: A New Approach.
. *M. Mirsalim, A. Doroudi, and J. S. Moghani* 1365
- Minimizing Torque Ripple in Permanent Magnet Synchronous Motors With Polymer-Bonded Magnets.
. *C. A. Borghi, D. Casadei, A. Cristofolini, M. Fabbri, and G. Serra* 1371
- Electrical Modeling of Transformer Connecting Bars *E. Clavel, J. Roudet, and A. Foggia* 1378
- Review of Dynamic Stability of Repulsive-Force Maglev Suspension Systems. *D. M. Rote and Y. Cai* 1383
- Microwave Properties of ZnTi-Substituted *M*-Type Barium Hexaferrites *Q. Feng and L. Jen* 1391
- Thermal Micromagnetic Model With a Pseudo-Three-Dimensional Junction for Magnetoresistive Heads.
. *L. Chen, J. Giusti, and J. Fernandez-de-Castro* 1395
- Micromagnetic Analysis of the Effect of Intergranular Exchange on Thermal Stability in Magnetic Recording (I):
Two-Dimensional Isotropic Longitudinal Media. *H. Zhou and H. N. Bertram* 1405
- Micromagnetic Analysis of the Effect of Intergranular Exchange on Thermal Stability in Magnetic Recording (II): Slightly
Oriented Longitudinal Media *H. Zhou and H. N. Bertram* 1417
- Micromagnetic Analysis of the Effect of Intergranular Exchange on Thermal Stability in Magnetic Recording (III):
Perpendicular Media *H. Zhou, H. N. Bertram, and M. E. Schabes* 1422
- Simulation of the Off-Track Capability of a One Terabit per Square Inch Recording System
. *Z. Jin, H. N. Bertram, B. Wilson, and R. Wood* 1429
- Ti_xZr_{1-x} Underlayers for CoCrPtB Perpendicular Magnetic Recording Media
. *Y.-N. Hsu, K. W. Wierman, B. Lu, T. J. Klemmer, and J. K. Howard* 1436
- Prediction of Track Misregistration Due to Disk Flutter in Hard Disk Drive
. *Y.-B. Chang, D.-K. Park, N.-C. Park, and Y.-P. Park* 1441

ERRATUM

- Correction to "Comparison of the Normal and Initial Induction Curves" *D. Bulte and R. Langman* 1447
-



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Two-Dimensional Cartesian Air-Gap Element (CAGE) for Dynamic Finite-Element Modeling of Electrical Machines With a Flat Air Gap

R. Wang, H. Mohellebi, T. J. Flack, M. J. Kamper, J. D. Buys, and M. Feliachi

Abstract—This paper extends the air-gap element (AGE) to enable the modeling of flat air gaps. AGE is a macroelement originally proposed by Abdel-Razek *et al.* for modeling annular air gaps in electrical machines. The paper presents the theory of the new macroelement and explains its implementation within a time-stepped finite-element (FE) code. It validates the solution produced by the new macroelement by comparing it with that obtained by using an FE mesh with a discretized air gap. It then applies the model to determine the open-circuit electromotive force of an axial-flux permanent-magnet machine and compares the results with measurements.

Index Terms—Air gaps, finite-element methods, linear motors, magnetic fields, mesh generation, permanent magnet generators.

I. INTRODUCTION

THE air-gap element (AGE) was originally proposed by Abdel-Razek *et al.* [1], [2] for the modeling of the annular air-gap region in electrical machines. The AGE replaces the finite-elements, which would normally be used to discretize the air gap, with a single macroelement. This macroelement is based on the analytical solution of the air-gap field. The main advantages of the AGE are: the ease with which relative motion between rotor and stator is treated (no need for remeshing the air gap); greater accuracy, since the air-gap field is related to its analytical solution; the avoidance of finite-elements with poor aspect ratios, or alternatively the use of a high peripheral density of nodes. The main disadvantage is that the resulting stiffness matrix has a large profile, especially if the number of nodes on the AGE boundary is large. This can lead to large CPU time, although a method for alleviating this problem is given in [3].

Since a typical rotating machine has an annular air gap, the AGE was initially derived for the polar coordinate system. The

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R. Wang and M. J. Kamper are with the Department of Electrical Engineering, University of Stellenbosch, Matieland 7602, South Africa (e-mail: rwang@ing.sun.ac.za; kamper@ing.sun.ac.za).

H. Mohellebi is with the Department of Electrotechnique, University of Mouloud MAMMERI, Tizi-Ouzou BP17RP, Algeria (e-mail: mohellebi@yahoo.fr).

T. J. Flack is with the Engineering Department, Cambridge University, Cambridge CB2 1PZ, U.K. (e-mail: tjf@eng.cam.ac.uk).

J. D. Buys is with the Department of Mathematics, University of Stellenbosch, Matieland 7602, South Africa (e-mail: jdb@land.sun.ac.za).

M. Feliachi is with the GE44- (LRTI-IUT3), CRTT, Bd de l'Université, 44602 Saint-Nazaire cedex, France (e-mail: mouloud.feliachi@ge44.univ-nantes.fr).

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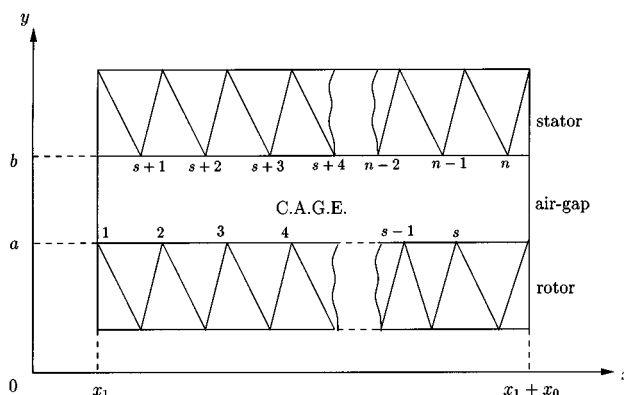


Fig. 1. Solution domain of the CAGE.

modeling of some special electrical machines such as the linear machine or the disc-type rotating machine requires a basic reformulation of the AGE, in order to deal with the rectangular shape of the air gap. Since this is associated with the Cartesian coordinate system, it is necessary to derive the AGE for Cartesian problems. This paper presents the development of a Cartesian air-gap element (CAGE). The new CAGE is validated by comparison with results obtained using the standard FE method, in which the air-gap is discretized. The application of the CAGE is also illustrated by a typical case study.

II. CARTESIAN AIR-GAP ELEMENT MODEL

Consider a generic linear machine, for which the airgap is as shown in Fig. 1. Clearly, in the air gap region there is no current, and the reluctivity is that of free space, ν_o . In Cartesian coordinates, the field in the airgap region is therefore governed by

$$\frac{\partial^2 A(x, y)}{\partial x^2} + \frac{\partial^2 A(x, y)}{\partial y^2} = 0 \quad (1)$$

in which $A(x, y)$ denotes the z -directed component of the magnetic vector potential. Like conventional electrical machines, special electrical machines such as the axial flux and the linear machine also exhibit structural periodicity which in turn leads to periodicity in the magnetic field distribution. In this paper, the periodicity is taken to be of the form

$$A(x, y) = A(x + x_o, y) \quad (2)$$

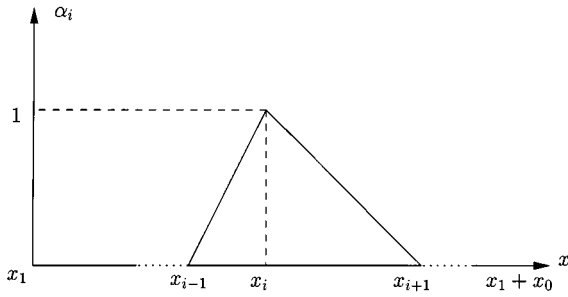


Fig. 2. Definition of function $\alpha_i(x, c)$.

where x_o is the period of the function $A(x, y)$. Applying the technique of separation of variables and enforcing the boundary condition of (2), the solution to (1) is

$$A(x, y) = F_o \cdot y + G_o + \sum_{n=1}^{\infty} (F_n \cdot e^{\lambda_n y} + G_n \cdot e^{-\lambda_n y}) \cdot (H_n \cos \lambda_n x + K_n \sin \lambda_n x) \quad (3)$$

where $\lambda_n = \pm 2n\pi/x_o$ and $F_o, G_o, F_n, G_n, H_n, K_n$ are constants which are to be determined.

In order to maintain the continuity of $A(x, y)$ at the transition between classical finite elements and the CAGE (i.e., the lines $y = a$ and $y = b$), the following boundary condition must be satisfied:

$$A(x, c) = \sum_{i=k}^l \alpha_i(x, c) \cdot A_i^b \quad (4)$$

where A_i^b is the nodal value of the magnetic vector potential at node i on the CAGE boundary, $\alpha_i(x, c)$ is the Lagrange polynomial defined from the shape functions of the adjacent classical elements, and

$$k = \begin{cases} 1 & \text{and } l = s \text{ if } c = a, \\ s+1 & \text{and } l = t \text{ if } c = b. \end{cases}$$

The nodal shape function $\alpha_i(x, c)$ for the first order triangular element is illustrated in Fig. 2. This may be represented as

$$\alpha_i(x, c) = \begin{cases} \frac{x - x_{i-1}}{x_i - x_{i-1}} & \text{if } x_{i-1} \leq x \leq x_i \\ \frac{x - x_{i+1}}{x_i - x_{i+1}} & \text{if } x_i \leq x \leq x_{i+1} \\ 0 & \text{if } x_1 \leq x \leq x_{i-1} \\ 0 & \text{if } x_{i+1} \leq x \leq x_1 + x_o. \end{cases} \quad (5)$$

To facilitate further mathematical manipulation, (5) is expanded into a Fourier series

$$\alpha_i(x, c) = a_{0i} + \sum_{n=1}^{\infty} [a_{ni} \cos \lambda_n x + b_{ni} \sin \lambda_n x] \quad (6)$$

where a_{0i}, a_{ni} and b_{ni} may be found as

$$\left\{ \begin{aligned} a_{0i} &= \frac{x_{i+1} - x_{i-1}}{x_o} \\ a_{ni} &= -\frac{4}{x_o} \cdot \frac{1}{\lambda_n^2} \left[\frac{1}{x_i - x_{i-1}} \sin \frac{\lambda_n(x_i + x_{i-1})}{2} \right. \\ &\quad \left. \sin \frac{\lambda_n(x_i - x_{i-1})}{2} + \frac{1}{x_i - x_{i+1}} \right. \\ &\quad \left. \sin \frac{\lambda_n(x_{i+1} + x_i)}{2} \sin \frac{\lambda_n(x_{i+1} - x_i)}{2} \right] \\ b_{ni} &= \frac{4}{x_o} \cdot \frac{1}{\lambda_n^2} \left[\frac{1}{x_i - x_{i-1}} \sin \frac{\lambda_n(x_i - x_{i-1})}{2} \right. \\ &\quad \left. \cos \frac{\lambda_n(x_i + x_{i-1})}{2} + \frac{1}{x_i - x_{i+1}} \right. \\ &\quad \left. \sin \frac{\lambda_n(x_{i+1} - x_i)}{2} \cos \frac{\lambda_n(x_{i+1} + x_i)}{2} \right]. \end{aligned} \right. \quad (7)$$

The constants in (3) may now be found by equating $A(x, a)$ and $A(x, b)$ obtained from (3) with the respective terms obtained by substituting (6) and (7) into (4). The final result is

$$A(x, y) = \sum_{i=1}^t \alpha_i^\varepsilon(x, y) \cdot A_i^b \quad (8)$$

where $\alpha_i^\varepsilon(x, y)$ is given by

$$\alpha_i^\varepsilon(x, c) = \frac{y - c}{c' - c} \cdot \frac{a_{0i}}{2} + \sum_{n=1}^{\infty} \frac{e^{\lambda_n(y-c)} - e^{\lambda_n(c-y)}}{e^{\lambda_n(c'-c)} - e^{\lambda_n(c-c')}} \cdot (a_{ni} \cos \lambda_n x + b_{ni} \sin \lambda_n x). \quad (9)$$

and

$$c = \begin{cases} a \text{ and } c' = b & \text{if } i \in \{1, 2, \dots, s\} \\ b \text{ and } c' = a & \text{if } i \in \{s+1, \dots, t\}. \end{cases}$$

The energy functional for the air gap, $[S]^\varepsilon$, is

$$F_\varepsilon = \frac{1}{\mu_o} \iint_{S_\varepsilon} \frac{B^2}{2} dS_\varepsilon. \quad (10)$$

By substituting for \mathbf{B} in (10) using $\mathbf{B} = \nabla \times \mathbf{A}$, where $A(x, y)$ is given by (8) and (9), and then differentiating with respect to A_i for $i = 1 \dots t$ gives

$$\frac{\partial F_\varepsilon}{\partial [A]^\varepsilon} = \frac{1}{\mu_o} [S]^\varepsilon [A]^\varepsilon \quad (11)$$

where the general term of the matrix $[S]^\varepsilon$ is

$$S_{ij}^\varepsilon = \frac{x_o}{4} \frac{b-a}{(c'-c)(f'-f)} a_{0i} a_{0j} + \frac{x_o}{2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \frac{1}{[e^{\lambda_n(c'-c)} - e^{\lambda_n(c-c')}] [e^{\lambda_n(f'-f)} - e^{\lambda_n(f-f')}]}$$

$$\cdot \left[\left(e^{\lambda_n(b-c)} - e^{\lambda_n(c-b)} \right) \left(e^{\lambda_n(b-f)} + e^{\lambda_n(f-b)} \right) - \left(e^{\lambda_n(a-c)} - e^{\lambda_n(c-a)} \right) \left(e^{\lambda_n(a-f)} + e^{\lambda_n(f-a)} \right) \right] \cdot (a_{ni} a_{nj} + b_{ni} b_{nj}). \quad (12)$$

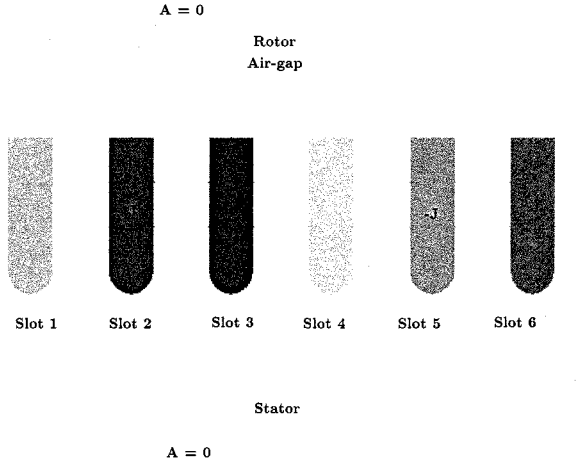


Fig. 3. Simple linear electrical machine model.

where

$$f = \begin{cases} b \text{ and } f' = a & \text{if } i \in \{1, 2, \dots, s\} \\ a \text{ and } f' = b & \text{if } i \in \{s+1, \dots, t\}. \end{cases}$$

III. COUPLING SCHEME

The implementation of the CAGE into the standard finite element program is carried out in the same way as for the AGE for annular air gaps. The energy-related functional is broken down into contributions due to the standard finite elements and the macroelement, F^e and F^ε , respectively. It is then minimized with respect to A

$$\frac{\partial F}{\partial [A]} = \sum_{e=1}^P \frac{\partial F^e}{\partial [A]^e} + \frac{\partial F^\varepsilon}{\partial [A]^\varepsilon}. \quad (13)$$

In order to reduce the time taken to compute the CAGE terms at different rotor positions, the scheme proposed in [3] was implemented. Close inspection of (12) and its counterpart for the annular AGE [1] reveals that this scheme can be directly applied to the CAGE with very little modification. In order to further reduce the CPU time taken, it was possible to utilize a negative periodic model, thus reducing the number of unknowns to be solved for by a factor of two approximately. The modification to the CAGE for negative periodic boundary conditions was done in the same manner proposed in [3]. This proves that the first term in (12) disappears, and the second term is doubled.

IV. VALIDATION BY COMPARISON

In order to validate the CAGE, the simplified linear problem shown in Fig. 3 was modeled. Firstly the airgap was discretized using two layers (120 nodes per layer) of classical first-order triangular elements, and the resulting FE model solved. Then, the CAGE was substituted in place of the discretized airgap, leaving the rest of the FE mesh unchanged, and solved. Homogeneous Dirichlet conditions were assigned to the top and bottom boundaries, whilst positive periodicity conditions were assigned to the left and right boundaries.

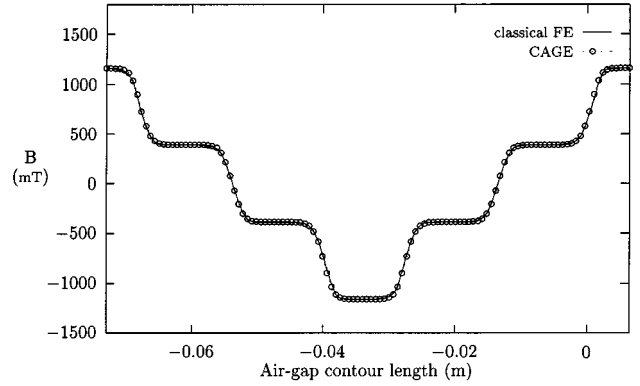


Fig. 4. Comparison of calculated air-gap flux density by using classical FE and CAGE.

TABLE I
COMPARISON OF THE CALCULATED MAGNETIC STORED ENERGY IN DIFFERENT SUBREGIONS OF THE MODEL

Regions	C.A.G.E. (μJ)	Classical FE (μJ)	Diff%
Slot1 (Cu)	0.519085776	0.519129716	0.01%
(air)	0.692833408	0.695610829	0.40%
Slot2 (Cu)	2.07633383	2.07650904	0.01%
(air)	2.69936816	2.69670033	0.09%
Slot3 (Cu)	0.519085782	0.519129719	0.01%
(air)	0.692830925	0.699608371	0.09%
Slot4 (Cu)	0.519085777	0.519129716	0.01%
(air)	0.692833425	0.699610829	0.09%
Slot5 (Cu)	2.07633377	2.07650904	0.01%
(air)	2.69936826	2.69670033	0.09%
Slot6 (Cu)	0.519085774	0.519129719	0.01%
(air)	0.692830919	0.695608317	0.40%
Stator iron	0.242891804	0.242665537	0.14%
Rotor iron	0.025792278	0.0254261082	0.14%
Air-gap	35.8157674	35.9925716	0.49%

Slots 1–6 were excited with balanced three-phase currents at an instant in time, and the airgap flux density distribution obtained by both methods is plotted in Fig. 4. It is evident that the result obtained using the CAGE agrees very well with that obtained using a discretized airgap.

Further examination and verification has also been performed by comparing the respective magnetic stored energies in various parts of the model obtained from both methods, as shown in Table I. It is clear that the results are in very good agreement. The worst discrepancy is about 0.5%.

V. APPLICATION

The two-dimensional finite-element (2-D FE) modeling of an iron-less stator AFPM machine [4] (illustrated in Fig. 5) is usually carried out by introducing a radial cutting plane, which is then developed into a 2-D flat model. Fig. 6(a) shows an FE mesh for such a model, which spans one pole of the AFPM machine. The mesh consists of 622 elements and 447 nodes, of which 138 nodes lie on the boundaries of the CAGE. The air-gap region was modeled using the CAGE. The model was time-stepped over a time interval of 16 ms (which corresponds to the rotor moving by one pole-pair), using a time-step of 0.2 ms. This required 80 FE solutions to be carried out. Fig. 6(b) shows the flux plot of the AFPM machine at a certain position.

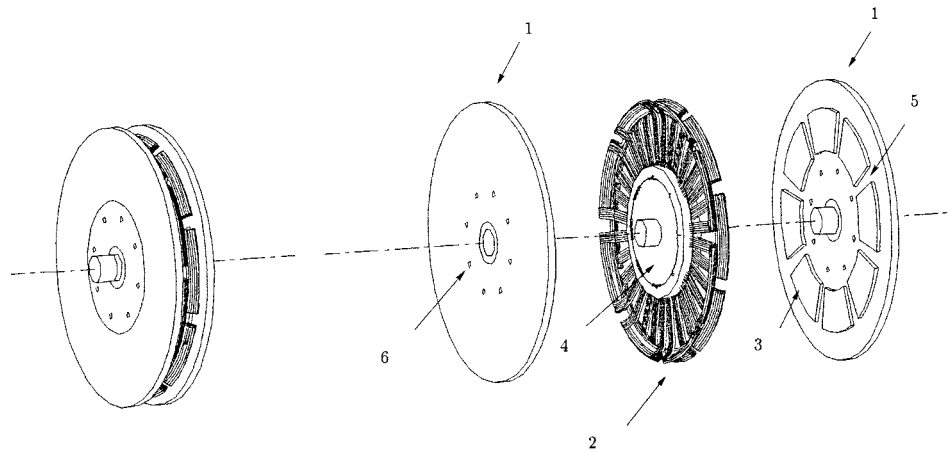


Fig. 5. Exploded view of an AFPM machine. 1: rotor discs. 2: stator winding. 3: permanent magnets. 4: epoxy core. 5: radial channels. 6: air-inlet holes.

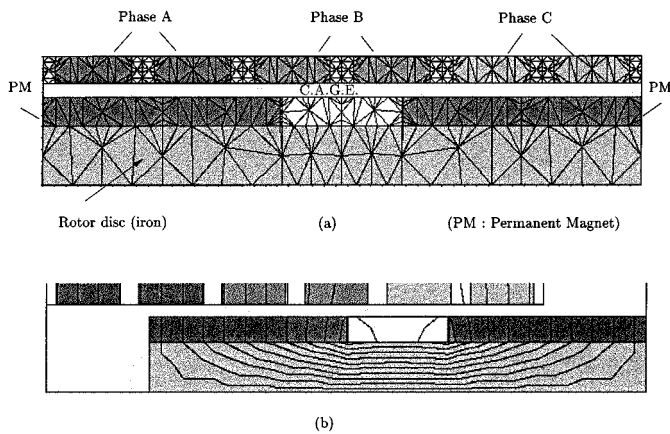


Fig. 6. Time-step FE model of an iron-less stator AFPM machine.

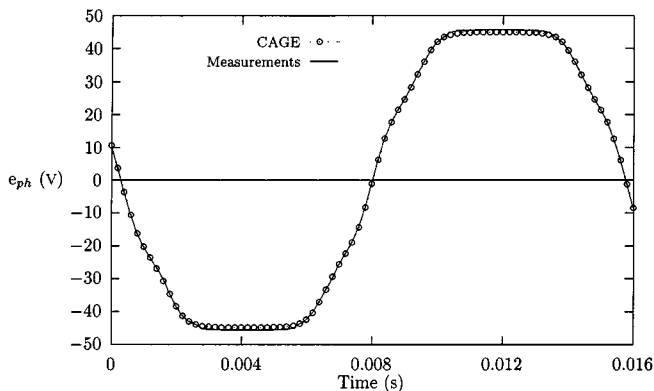


Fig. 7. Phase EMF as a function of time at 970 rpm.

At every rotor position, the total phase flux linkages were deter-

mined from the FE field solution, from which the phase electromotive force (EMF), $e(t)$, could be found as

$$e(t) = \frac{d\lambda}{dt} = \frac{\lambda(\theta_2) - \lambda(\theta_1)}{t_2 - t_1}. \quad (14)$$

The total CPU time for this simulation was 107 s on a 333-MHz PC running the Redhat Linux operating system. The calculated EMF induced in the stator phase winding at a shaft speed 970 rpm is compared with measurement in Fig. 7. It is seen that the correlation between calculated and measured EMF is very good.

VI. CONCLUSION

This paper has reported the development of a 2-D CAGE, and validated it by comparison with the standard FE method. It has also shown that the CAGE is still a viable method for time-stepped FE modeling of electrical machines, and the method has been successfully applied to an AFPM machine. Measured and simulated results of the AFPM machine open-circuit EMF when driven at fixed speed were found to be in very good agreement. The CAGE formulation derived here is applicable to most linear machines and some other special machines such as rotating disc machines.

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