Cogging Torque Sensitivity in Design Optimisation of Low Cost Non-overlap Winding PM Wind Generator

Johannes H.J. Potgieter and Maarten J. Kamper

Abstract — Low cost and low cogging torque are the most important aspects in the design of direct drive PM wind generators. In this paper a low cost PM wind generator with an irregular, parallel slotted stator is analysed with respect to cogging torque. The sensitivity of average torque and cogging torque to machine dimension variations are investigated. From this a method is proposed whereby regions of low cogging torque can be identified more quickly in the design optimisation. Finite element analysis is used to determine the cogging torque. Calculated results are validated by practical measurements on a 15 kW PM wind generator.

Index Terms—Permanent magnet, wind generator, cogging torque, design optimisation, sensitivity analysis.

I. INTRODUCTION

Torque quality and cost are the two most important aspects to consider in the design of direct drive permanent magnet (PM) wind generators. These aspects are even more important than efficiency and average torque performance, and generator-mass.

Torque quality refers to the torque ripple generated by the machine, which mainly are caused by stator winding MMF harmonics and the slotted air gap of the machine; at no-load the slotted air gap causes the so-called cogging torque of the machine. Torque ripple can be a source of serious vibration and acoustic noise in PM wind generator systems. More over, cogging torque can lead to complete failure of start-up as the wind turbine generate very low torque at low turbine speeds. Improving thus, the torque quality of the PM wind generator to the lowest percentage torque ripple is of the utmost importance.

The torque quality of PM electrical machines is a topic that has received extensive research and attention in literature. Several techniques exist to reduce the torque ripple and cogging torque. These techniques include skewing the magnets and stator slots, varying the magnet positions, closing stator slots, and introducing auxiliary slots and teeth [1]–[9]. Manufacturing and assembling tolerances in using these techniques, however, require sensitivity analysis on the dimensions of the machine [4], [9]. Furthermore, most of these techniques, due to constructional complexities, are not suitable when opting for a low-cost wind generator solution.

Reducing the cost of manufacturing is one of the largest driving forces in the production of PM wind generators. This can be attributed to a highly competitive market, but also to the fact that wind generator systems are still relatively expensive. An important aspect in reducing cost is to reduce the labour cost to manufacture the machine.

The focus of this paper, thus, is to consider a low cost and low cogging torque PM wind generator design. The intend is to present a more visual representation of the PM electrical machine behaviour with regard to cogging torque. The cogging effects imposed by several machine design parameters are investigated to enable a broad interpretation of machine cogging torque characteristics.

Obviously the average torque and efficiency performance of the PM wind generator must also be considered in the design. The aim in the design optimisation, thus, is to maximise the torque output of the machine subject to minimum cogging torque. While maximisation of the average torque by means of finite element (FE) analysis and optimisation algorithms is no more complex [10], the minimisation of cogging torque using this process is not so easy as is shown in the paper.

II. MACHINE SELECTION AND OPTIMISATION PARAMETERS

The analysis in this paper is based on a case-study of a 15 kW direct drive PM wind generator. The performance of this generator is at rated values of 150 r/min, 50 Hz and 400 V. The pole number of the generator is \(p = 40\).

A. Low Cost PM Machine Selection

With the emphasis on low cogging torque and low cost a non-overlap winding PM machine is selected, which has lower cogging torque and also less number of coils than the overlap winding PM machine. To reduce labour cost a single layer winding is chosen that has half the number of coils than the double layer winding. A single layer winding also has the advantage that the slot pitch can be varied to reduce the cogging torque. An important disadvantage of a single layer winding, however, is the large sub MMF harmonic that severely increase the eddy current losses in the magnets and rotor yoke if solid. Methods to reduce these losses effectively are proposed in [11]; further consideration of these losses is beyond the scope of this paper. Further to reduce labour cost, pre-wound coils must be used that requires open and pairs of parallel stator slots.

With \(p = 40\) there are three options of single layer pole-slot combinations with high winding factors, that can be selected for the PM wind generator. These are the 40/36, 40/42 or 40/48 combinations [12]. The 40/42 combination with only one winding section \((W_x = 1)\) [13] is immediately discarded due to the unbalanced magnetic field in the air gap under load. The 40/36 combination is also a question in this regard with a very low number of winding sections namely \(W_x = 2\). The 40/36 option has less number of coils compared to the 40/48 combination, but the ratio of the open-slot opening to the magnet pitch is larger, which is a disadvantage, and the winding factor is also lower. Another option is to look at the 42/36 combination, with the rated turbine speed then slightly lower. The slot opening to magnet pitch ratio of the 42/36 combination, however, is...
even worse than the 40/36 combination.

Hence, the 40/48 combination is selected with \( W_s = 8 \). In the FE modelling negative boundary conditions can be used [13] and only five poles and six slots have to be meshed. A cross-section of the FE model of the single layer, open and paralleled slots, low cost PM generator proposed in the paper is shown in Fig. 1a. To investigate the effects caused by the irregular, parallel slotted stator, a comparison is done with a regular taper slotted PM machine as shown in Fig. 1b. Some design detail and explanation of the PM wind generator that was build and tested are given in Table I and Fig. 1c respectively.

![Image](Fig_1.png)

**Table I: PM Wind Generator Parameters.**

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator inner diameter, ( d_i ) (mm)</td>
<td>494</td>
<td>Magnet material</td>
<td>N48H</td>
</tr>
<tr>
<td>Stator yoke height, ( h_r ) (mm)</td>
<td>10</td>
<td>Number of stator slots, ( S )</td>
<td>48</td>
</tr>
<tr>
<td>Stator outer diameter (mm)</td>
<td>623</td>
<td>Number of turns per coil</td>
<td>424</td>
</tr>
<tr>
<td>Slot width, ( \sigma_s )</td>
<td>0.44</td>
<td>Number of poles, ( p )</td>
<td>40</td>
</tr>
<tr>
<td>Slot pitch, ( \sigma_t )</td>
<td>1.01</td>
<td>Rated voltage (V)</td>
<td>408</td>
</tr>
<tr>
<td>Axial length of stack, ( \ell ) (mm)</td>
<td>100</td>
<td>Rated current (A)</td>
<td>23</td>
</tr>
<tr>
<td>Rotor inner diameter (mm)</td>
<td>627</td>
<td>Rated output power (kW)</td>
<td>15</td>
</tr>
<tr>
<td>Rotor outer diameter, ( d_o ) (mm)</td>
<td>653.5</td>
<td>Rated torque (Nm)</td>
<td>1000</td>
</tr>
<tr>
<td>Rotor yoke height, ( h_m ) (mm)</td>
<td>7.25</td>
<td>Power factor</td>
<td>0.95</td>
</tr>
<tr>
<td>Rotor magnet height, ( h_m )</td>
<td>6</td>
<td>Efficiency (%)</td>
<td>94.4</td>
</tr>
<tr>
<td>Magnet to pole pitch ratio, ( \sigma_m )</td>
<td>0.73</td>
<td>Rated turbine speed (r/min)</td>
<td>150</td>
</tr>
</tbody>
</table>

**B. Design Optimisation Parameters**

The machine dimensions that affect the cogging torque are the magnet pitch, slot pitch, the slot opening width and the rotor and stator yoke heights as explained in Fig. 2. The former three dimensions are rather expressed in per unit values and are calculated as the magnet pitch to the pole pitch ratio, \( \sigma_m \), the slot pitch to the average slot pitch ratio, \( \sigma_t \), and the slot opening width to the average slot pitch ratio, \( \sigma_g \). Mathematically these ratios are calculated from angular dimensions as

\[
\sigma_m = \frac{\theta_m}{\theta_p}; \quad \sigma_t = \frac{\theta_t}{\theta_s}; \quad \sigma_g = \frac{\theta_g}{\theta_s},
\]

where

\[
\theta_m = \frac{2\pi}{S}; \quad \theta_p = \frac{2\pi}{p},
\]

and \( S \) the number of stator slots. The parameters, thus, that have to be optimised to minimise the cogging torque are given in matrix-format by \( X_1 \) in (3). The other machine dimensions to be optimised are given by \( X_2 \) in (3).

\[
X_1 = \begin{bmatrix} \sigma_m \\ \sigma_t \\ \sigma_g \\ h_r \\ h_s \end{bmatrix}, \quad X_2 = \begin{bmatrix} d_o \\ d_i \\ \ell \\ h_m \end{bmatrix}, \quad U = \begin{bmatrix} \tau_{ave} \\ \Delta \tau \end{bmatrix}.
\]

The performance parameters in \( U \) of (3) are calculated in per unit values as

\[
\tau_{ave} = \frac{\tau_{ave}}{\tau_{rated}}; \quad \Delta \tau = \frac{\Delta \tau_{cog}}{\tau_{rated}},
\]

where \( \tau_{ave} \) and \( \Delta \tau_{cog} \) are the average and peak-to-peak ripple torque respectively of the machine and \( \tau_{rated} = 1000 \) Nm from Table I.

**III. DESIGN OPTIMISATION METHOD**

The design optimisation method proposed in the paper is to first optimise \( X_1 \) and \( X_2 \) of (3) in terms of the average torque performance, \( \tau_{ave} \), of the machine and then to optimise \( X_1 \) a second time to minimise \( \Delta \tau \). In the same way as described in [10], FE analysis together with an optimisation algorithm are used to optimise \( X_1 \) and \( X_2 \), by maximising the torque per copper losses of the machine and keeping the active mass to a minimum subject to a torque constraint of \( \tau_{ave} > 1.0 \) pu. In this design optimisation the copper loss was set at \( P_{cu} = 600 \) W according to the cooling capacity of the wind generator. The outer diameter was set at a maximum value of 667 mm and the axial length was limited to \( \ell = 100 \) mm. The outcome of this design optimisation according to the parameters of (3) is given as

\[
X_1(\tau_{ave}) = \begin{bmatrix} 0.98 \\ 1.0 \\ 0.53 \\ 14.0 \\ 8.0 \end{bmatrix}, \quad X_2(\tau_{ave}) = \begin{bmatrix} 667 \\ 496 \\ 100 \\ 1.13 \\ 0.26 \end{bmatrix}.
\]

It is clear from (5) that the cogging torque is far to high (26 %) for the PM wind generator. The cogging torque is calculated by the Maxwell’s stress tensor method, and by position-stepping the rotor till a peak-to-peak torque, \( \Delta \tau_{cog} \), is obtained. All results are calculated from magneto static FE solutions. Using static solutions, transient effects do not influence the cogging torque results.
IV. Effect of Machine Dimensions on Torque

A. Single parameter variation

The three parameters in (3) having the largest effect on the cogging torque are $\sigma_m$, $\sigma_t$, and $\sigma_g$. The torque behaviour that results from these parameter variations, thus, needs to be thoroughly analyzed. This is done by keeping all machine design parameters and $P_{cu}$ fixed, while varying separately each of the above parameters. The effect of these variations on $\tau_{ave}$ and $\Delta \tau$ of the machine of Fig. 1a are shown in Figs. 2 – 4. The much more sensitive nature of $\Delta \tau$ compared to $\tau_{ave}$ can be clearly seen. It is clear that the best cogging torque performance occurs at specific values of $\sigma_m$, $\sigma_t$, and $\sigma_g$. Due to the much less effect on $\tau_{ave}$, these values can be chosen as the optimum design parameters without affecting the overall machine performance significantly.

Fig. 2. Per unit cogging torque and average torque versus magnet pitch of the irregular parallel slotted machine (Fig. 1a).

![Fig. 2](image1)

Fig. 3. Per unit cogging torque and average torque versus slot pitch of the irregular parallel slotted machine (Fig. 1a).

![Fig. 3](image2)

Fig. 4. Per unit cogging torque and average torque versus slot width of the irregular parallel slotted machine (Fig. 1a).

![Fig. 4](image3)

B. Multi-parameter variation

Due to the irregular and sensitive nature of $\Delta \tau$, a more thorough analysis is done through multi-parameter variation with the results shown in Figs. 5 – 7. A more visualised representation of the machine’s torque behaviour is obtained in this way. Both Figs. 5 and 6 are done for the irregular parallel slotted machine. In order to observe the effect caused by the irregular slotting, Fig. 7 gives the cogging torque results for the regular taper slotted machine.

It is clear that the regular taper slotted machine has a much smoother and overall lower cogging torque than the irregular parallel slotted machine; this is especially true when comparing Figs. 5a and 7a. However, Figs. 5 and 6 show that low cogging torque values can be obtained for the irregular parallel slotted machine. What is interesting is that there are valleys of low cogging torque which are very much independent of the magnet pitch. Again the smooth and less sensitive nature of the average torque are clear from Figs. 5b and 6b.

Figs. 5 – 7 give valuable information about the machine’s torque behaviour and gives a clear indication on what parameter values should be chosen. However, to obtain these plots, intensive and time consuming FE simulations are needed.

To investigate this aspect further, Figs. 8 and 9 shed more light on the optimum parameter regions for best cogging torque performance. Fig. 8 corresponds to Fig. 5a where the cogging torque is plotted versus the slot width with the magnet pitch a parameter. Fig. 9 corresponds to Fig. 6 where the cogging torque is plotted versus the slot pitch with the magnet pitch a parameter.

Fig. 5. (a) Per unit cogging torque and (b) average torque versus slot width and magnet pitch for the irregular parallel slotted machine (Fig. 1a).

![Fig. 5](image4)
It is clear from Figs. 8 and 9 that there are slot width regions and slot pitch regions where the cogging torque is very much independent of the magnet pitch. Hence, only these regions need to be investigated comprehensively. A decent performing machine with low cogging torque can, thus, be obtained, while still staying within acceptable FE simulation times.

C. Effect of stator and rotor yoke heights.

Despite the dominant cogging effects caused by $\sigma_m$, $\sigma_t$, and $\sigma_g$, changes in the stator and rotor yoke heights imposed significant cogging torque variations as well. This is shown in Fig. 10, which shows the cogging torque versus the rotor yoke height with the stator yoke height a parameter; Fig. 11 shows the effect of this variation on the average torque.

Again the sensitive nature of $\Delta \tau$ and the smooth, lesser variation of $\tau_{ave}$ are seen. With this optimisation the reduction in the generator mass can be substantial; it can be seen from Fig. 10 that lowest cogging torques occurs at much thinner rotor yokes than the optimum results of (5).

Another parameter change investigated, is a change in lamination steel. Fig. 12 compares the cogging torque waveforms for two different electrical machine lamination steels. A clear difference can be observed.

D. Selection of Optimum values

In (6) the optimum parameter values are given for the PM generator that gives the best cogging torque performance. Comparing the performance of the minimum cogging torque design of (6) with the performance of the maximum torque design of (5), there is a drop in average torque of 13 %, but a cogging torque reduction from 26 % to 0.2 %. Note that the active mass of the design of (6) is significantly less.
Fig. 9. Per unit cogging torque versus slot pitch with magnet pitch a parameter of the irregular parallel slotted machine (Fig. 1a).

Fig. 10. Per unit cogging torque versus rotor yoke height with stator yoke height a parameter of the irregular parallel slotted machine (Fig. 1a).

Fig. 11. Per unit average torque versus rotor yoke height with stator yoke height a parameter of the irregular parallel slotted machine (Fig. 1a).

Fig. 12. Cogging torque waveforms of the prototype PM wind generator for two different lamination steels.

VI. CONCLUSION

A low cost, low cogging torque PM wind generator with a 10/12 pole-slot combination and parallel open stator slots is proposed and investigated in this paper. It is shown that the average generated torque shows low sensitivity (± 6 % of rated torque) to dimension variations in the search for minimum cogging torque. This validates the approach to first do a design optimisation based on average values and subject to certain constraints, and then to minimise the
cogging torque by final optimisation of some of the machine dimensions.

Minimising the cogging torque is shown to require a high number of FE solutions to avoid local minimum function values. The minimum cogging torque region, however, can be quickly found by setting the slot pitch equal to 1.0 per unit, choose a typical magnet pitch and identify the per unit slot width region where the cogging torque is a minimum. This method is based on the finding that there are low cogging torque regions (valleys) where the cogging torque is fairly independent of the magnet pitch.

An interesting finding is the effect the yoke heights and material type have on the cogging torque. A significant reduction in cogging torque can be obtained by further adjustment of the yoke dimensions. Together with this is the finding that the irregular, parallel slotted layout has in general a higher cogging torque than the regular taper slotted layout. All these results clearly indicate that magnetic saturation in the back iron parts has a significant effect on the cogging torque of the machine.

With regard to manufacturing and assembly tolerances it is very important in the minimising of the cogging torque to optimise the dimensions in regions where there is low sensitivity to magnet pitch variations. It is much more difficult to maintain high manufacturing accuracy in the magnet dimensions and placing of the magnets, than in the manufacturing accuracy of the lamination dimensions.

VII. REFERENCES


VIII. BIOGRAPHIES

Johannes H.J. Potgieter (S’08) was born in Oudtshoorn, South Africa, in March 1985. He received the B.Eng. degree in electrical and electronic engineering from the University of Stellenbosch, Matieland, South Africa in 2007. He is currently working towards the completion of the M.Sc.(Eng.) degree at the Department of Electrical and Electronic Engineering at the University of Stellenbosch. His current research focus are wind power generation solutions and the optimising of permanent magnet machine technologies, including computer-aided design.

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