Classification of Symmetrical Non-Overlapping Three-Phase Windings

Johannes J. Germishuizen and Maarten J. Kamper

***Abstract*—Concentrated windings are commonly associated with permanent magnet machines. However, it is the opinion of the authors that this name does not uniquely describe this winding type. The paper suggests a unique classification scheme which is based on classical winding properties. This paper points out that non-overlapping windings can either be concentrated or distributed.**

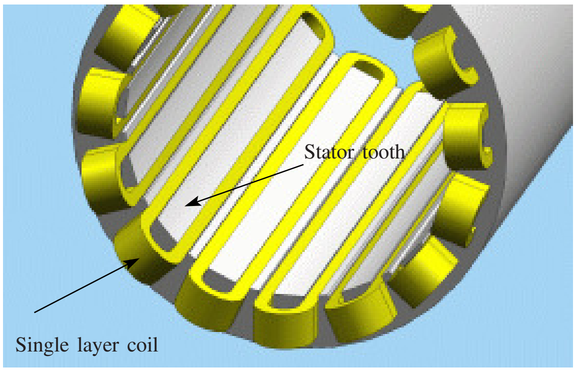
***Index Terms*—Non-overlapping, concentrated winding, basic winding, classification**

I. Introduction

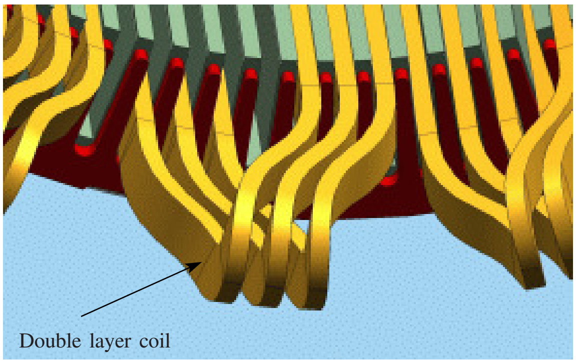
Analysis of electrical machines is a very well-known subject in electrical engineering and at a first glance one might think that this field cannot be investigated any further. However, this is not the case, since advances in materials such as permanent magnets offer new possibilities which might not have been economical just a few years ago. A typical example of how advances in other fields lead to new ideas in electrical engineering, and especially electrical machines, is permanent magnet motors with non-overlapping windings. Furthermore, this winding type is not uniquely defined and needs reformulation as mentioned by the authors in [1].

In recent years various authors have pointed out the advantages and explained the design of permanent magnet machines with non-overlapping end-windings e.g. as given in [2]. Fig. 1(a) shows a single layer non-overlapping winding (the end-windings do not overlap). In the drawing it is clear that each coil is wound around a stator tooth. Moreover, the coils do not overlap. Cros and Viarouge [3] certainly aroused interest in non-overlapping windings with their valuable paper entitled *Synthesis of High Performance PM Motors With Concentrated Windings*, since this is a paper which is very often used as a reference for these winding types. The authors use the term *concentrated windings* in the title and give examples of windings that are not in conformance with the classical definition of a concentrated winding. In their paper it becomes clear that coils wound around a stator tooth are meant. This could be a possible explanation for the use of the term "concentrated windings" rather than "tooth coil windings". It is interesting to note that in German this winding type is known as *Zahnspulen* which translated means tooth windings.

Fig. 1(b) shows a classical double layer winding with a coil pitch greater than one. Some of the coils are removed which makes it easier to identify the two layers. In this winding it is evident that the coil end-windings do overlap.



(a) Single layer non-overlapping winding



(b) Double layer overlapping winding

Fig. 1: Overlapping and non-overlapping coils

The importance of the theory on stator windings can be traced back to the 1930's. In the first of his two remarkable papers Vilém Klíma (Wilhelm Kauders) [4] explains the systematics of stator windings and the calculation of the winding factors. Also in this paper the induced voltage in the coil sides is already mentioned and represented as a vector. The resultant vector diagram was called the star of coil groups. Two years later, in the second paper by Klíma [5], the algebraic methods developed in the first paper were visualised by means of Tingley's diagram. The latter could be referred to as a linear representation of the star of slots that it is now called. The latter is also used by various authors such as are listed in [6]. The authors in [7] mention that Klíma's equation for the closed form of the distribution factor[1] of fractional slot windings is not found in textbooks.

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[1]Also known as the breadth factor.

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This paper will show that the winding in Fig. 1(a) is commonly referred to as a concentrated winding. It is the opinion of the authors that this name does not uniquely describe this winding type. Formally, a *concentrated winding* is one where the number of slots per pole and phase equals one. In this case the coil pitch equals the pole pitch and is referred to as a full-pitch concentrated winding as explained in textbooks such as [8]. Therefore, the objective of this paper is to answer the following question:

> *How can concentrated windings commonly associated with permanent magnet machines be classified in a unique way?*

In order to answer this question the paper suggests a unique classification scheme which is based on the classical winding properties, i.e. slots per pole and phase and coil pitch. In the first part of the paper the winding properties are explained, while the second part focuses on characteristics required in the winding design.

II. Definitions and terminology

In order to classify symmetrical non-overlapping three-phase windings in a unique way it is useful to have a coarse classification of symmetrical windings. Since symmetrical windings comprise such a great variety, an attempt to categorise them will depend on the properties which are used to sort them. These properties are explained in this section. The characteristic numbers give useful information on the winding when they are written as a reduced improper[2] fraction. Good references are given by Wach in [9] and [10].

A. Slots and coils per pole and phase

Any three-phase winding could be characterised by its number of slots per pole and phase. However, when comparing different winding designs with each other this number alone is insufficient. It does not take into account the number of layers the winding has. In addition, the number of coils per pole and phase should be defined. For a machine with Q_s stator slots and p pole pairs the following definition holds for the slots per pole and phase:

$$q = \frac{Q_s}{m\ 2p} = \frac{q_n}{q_d} \qquad (m = 3) \tag{1}$$

where $\frac{q_n}{q_d}$ is the reduced form of q. For the case where q_d equals one, q is an integer and the winding is called an integral[3] slot winding. When q_d is greater than one, it is called a fractional slot winding. The reduced form of q can be interpreted as follows: Each phase has q_n slots that are distributed over q_d poles.

The second winding property used in the classification scheme is the number of coils per pole and phase (not commonly used in literature). The definition is:

$$q_c = \frac{Q_c}{m\ 2p} = \frac{q_{c_n}}{q_{c_d}} \qquad (m = 3) \tag{2}$$

where $\frac{q_{c_n}}{q_{c_d}}$ is the reduced form of q_c. If q_{c_n} (in the reduced form of q_c) is greater than one, then it is called a distributed winding. If q_{c_n} equals one then it is a concentrated winding. The reduced form of q_c can be interpreted as follows: Each phase has q_{c_n} coils distributed over q_{c_d} poles.

B. Average coil pitch

The coil pitch is defined as the peripheral angle between the two coil sides. It is practical to express the coil pitch in terms of the number of slots. Therefore, it is an integer number. However, the average coil pitch can be a real number and is defined as:

$$y_p = \frac{Q_s}{2p} \qquad (y_p \in \mathbb{R}) \tag{3}$$

In the case where the average coil pitch as expressed in (3) equals a real number, the actual (or practical) coils pitch y_d must be an integer, i.e.

$$y_d = \text{int}(y_p) \pm k \qquad \begin{cases} k \in \mathbb{N} \\ y_d \geq 1 \end{cases} \tag{4}$$

When $y_d = y_p$ it is a full-pitch winding and if $y_d \neq y_p$ it is called a fractional pitch winding, i.e. the winding is chorded. The properties of single and double layer[4] windings are summerised in Table (I).

TABLE I: Winding properties

Single layer	$q = 2q_c$ $Q_s = 2Q_c$
Double layer	$q = q_c$ $Q_s = Q_c$
Average coil pitch Actual coil pitch	y_p as defined in (3) y_d as defined in (4)
Q_s – number of stator slots Q_c – number of winding coils q – number of slots per pole and phase, (1) q_n – numerator of q q_d – denominator of q q_c – number of coils per pole and phase, (2) q_{c_n} – numerator of q_c q_{c_d} – denominator of q_c	

C. Definition of the working harmonic

All harmonics are referred to in terms of the bore 2π of the machine, i.e. the *fundamental harmonic* has the order $\nu = 1$ and forms one pole pair. The harmonic that produces the magnetic field that interacts with the rotor poles $2p$ has the order $\nu = p$ and is called the *working harmonic*. Any other harmonic of the ν^{th} order will have ν pole pairs and spans a peripheral angle of $\frac{2\pi}{\nu}$ as explained in [13]. Often the harmonic orders are normalised with respect to the *working harmonic*, i.e. $\xi_{\nu/p}$. This then means that the *working harmonic* is written as ξ_1 and sub-harmonics will have a fraction as subscript.

[2] The reduced fraction could be either a proper or an improper fraction. An improper fraction has the numerator greater than the denominator.

[3] The authors in [11] refer to an integer slot winding while the authors in [12] use the term integral slot winding.

[4] A single and double layer winding have one and two coil sides per slot respectively.

D. Basic winding

The smallest repetitive segment is called the basic winding (as defined in [13]). Due to symmetry only the basic winding needs to be determined. If q_d is less than p the winding is composed of t identical *basic windings*, i.e.

$$t = \begin{cases} \gcd(Q_s, p) & \text{for double layer} \\ \gcd\left(\frac{Q_s}{2}, p\right) & \text{for single layer} \end{cases} \tag{5}$$

and gcd is called the greatest common divisor. In the case where $t = 1$ the winding has no symmetry. Each of the t *basic windings* will have Q_b slots and p_b pole pairs, therefore

$$Q_b = \frac{Q_s}{t} \quad \text{and} \quad p_b = \frac{p}{t} \tag{6}$$

The number p_b is the reduced pole pair. Another way of obtaining this number is by means of the denominator of q_c, i.e.

$$p_b = \begin{cases} \frac{1}{2} q_{c_d} & q_{c_d} \text{ even} \\ q_{c_d} & q_{c_d} \text{ odd} \end{cases} \tag{7}$$

which is independent of t. Examining (5), it can be seen that t can be rewritten as the gcd between the number of coils and the pole pairs, i.e.

$$t = \gcd(Q_c, p) \quad \begin{cases} Q_c = Q_s & \text{double layer} \\ Q_c = \frac{1}{2} Q_s & \text{single layer} \end{cases} \tag{8}$$

which is valid for both single and double layer windings. Although t is usually used as a variable for time it is commonly found in literature and will be used in the same way. Since the winding design is independent of time, it does not cause any confusion.

E. Winding symmetry

For a winding to be symmetrical the number of coils used in each of the phases must be equal. Therefore the quotient between Q_c and m must be an integer. A requirement for a winding to be symmetrical can be derived from (6). The symmetry condition can be expressed as

$$\frac{Q_s}{t} = mk, \quad k \in \mathbb{N} \tag{9}$$

which refers to the pole number, number of stator slots and phase number to each other. A very useful function employed in the method is the modulo function which determines the remainder after division, i.e. $\text{mod}(a, b) = a - \text{floor}(\frac{a}{b})$[5]. When using the modulo function it means that $\text{mod}(\frac{Q_s}{t}, m)$ must equal zero. There are different ways of deriving the constraints for the winding symmetry condition. Table II gives different variations found in literature to express the symmetry condition.

[5]The floor function of a real number x, floor(x), is a function whose value is the largest integer less than or equal to x.

TABLE II: Constraints for winding symmetry

Reference	Constraint
Wach, [9]	$\gcd(q_{c_d}, m) = 1$
	$\text{mod}\left(\frac{Q_c}{q_{c_n}} q_{c_d}, m\right) = 0$
Cross and Viarouge, [3]	$\text{mod}\left(\frac{Q_s}{\gcd(Q_s, 2p)}, m\right) = 0$
Bianchi, [6]	$\text{mod}\left(\frac{Q_s}{t}, m\right) = 0$

F. Reduced number of pole pairs

The lowest harmonic generated by a winding is given by $t = \gcd(Q_c, p)$ and the working harmonic equals p. The reduced pole number gives information on the subharmonics which are summarised as follows:

$$\begin{aligned} t &= p \quad \text{the winding has no subharmonics} \\ t &< p \quad \text{the winding has subharmonics} \end{aligned} \tag{10}$$

The reduced number of pole pairs can be calculated in two different ways. Two greatest common divisors, i.e. $\gcd(Q_c, 2\,m\,p)$ and $\gcd(Q_c, p)$, are used to get q_{c_d} and p_b respectively. The relationship between these two factors is as follows:

$$\gcd(Q_c, p) = \frac{\gcd(Q_c, m\,2p)}{r} \qquad r = \begin{cases} m & q_{c_d} \text{ even} \\ 2m & q_{c_d} \text{ odd} \end{cases} \tag{11}$$

III. Classification scheme

The aim of the classification scheme is to find a way to relate different winding types to each other. It is also a useful guideline to compare windings that belong to the same category. The following winding parameters are chosen for the classification:

1) the reduced form of the number of coils per pole per phase;
2) the average coil pitch; and
3) the number of layers.

Fig. 2 shows a possible way to classify symmetrical three-phase windings. The reason for choosing the average coil pitch as a property arises from the construction of the coils and it is valid for all types of windings. It could be seen as the ideal coil pitch. Additionally, the number of layers is a key parameter since the technology for manufacturing and the material used in single layer windings are different from that used in double layer windings. Using this classification scheme, the following definitions are associated with windings:

- *Overlapping and non-overlapping*: In overlapping windings the coil end-windings overlap and the coil pitch y_d is greater than one. If the coil pitch y_d equals one, the coil end-windings do not overlap.
- *Single and double layer*: These windings are differentiated by the number of coils compared to the number of stator slots. In single layer windings the number of coils equals half the number of stator slots, while for double layer windings the number of coils is equal to the number of stator slots.
- *Full-pitch and fractional pitch*: If the coil pitch y_d equals the average coil pitch it is called a full-pitch winding. If

$y_d \neq y_p$ the winding is chorded and is called a fractional pitch winding. In the special case where the number of slots per pole and phase equals one it is called a full-pitch concentrated winding.

- *Integral and fractional slot*: If the denominator q_d equals one it means that q is an integer and it is called an integral slot winding. If q_d is greater than one it means q is a fraction and hence a fractional slot winding. In addition, the average coil pitch y_p is a fraction.
- *Distributed and concentrated windings*: If the numerator q_{cn} of q_c is greater than one, the winding is distributed. This means that the coil sides are distributed over q_{cn} slots. The opposite of a distributed winding is a concentrated winding. In the case where q_{c_n} equals one, it is called a concentrated winding.

Fig. 2 defines the concentrated windings commonly associated with permanent magnet machines in a unique way, i.e. non-overlapping. Furthermore, it is shown that it could be either concentrated or distributed.

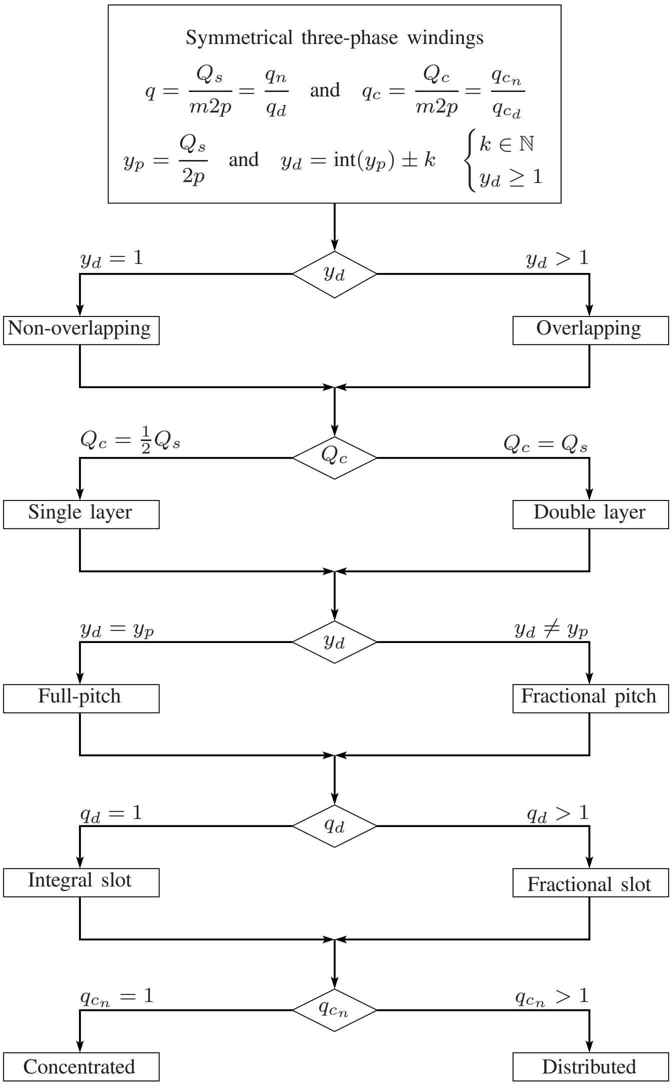


Fig. 2: Winding classification scheme

IV. Non-overlapping and overlapping windings

Fig. 3 summaries the four primary winding types. The two main categories are overlapping and non-overlapping. Each category can then be a single or double layer winding. Especially the winding types in Fig. 3(a) and Fig. 3(b) are commonly used in permanent magnet machines. However, authors use different terminologies and this can often be misleading. The most common terms found in literature for non-overlapping windings are:

- concentrated windings [14];
- concentrated fractional pitch windings [15];
- fractional slot wound [16]; and
- fractional slot [17].

In each case non-overlapping windings are meant, i.e. the winding type where the coils are wound around a stator tooth.

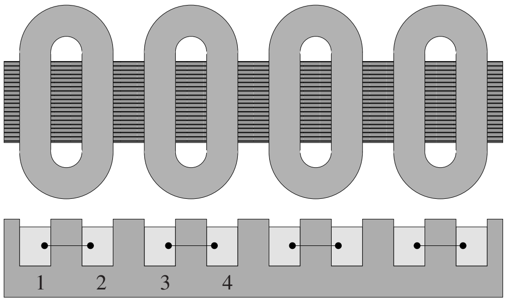
Table III gives a few examples where the classification scheme that has been introduced is applied. Examples 1 through 3 are typical single and double layer windings and clearly indicate the difference between integral and fractional slot. Examples 4 through 6 are commonly referred to in the literature as concentrated windings. It is the opinion of the authors that the term concentrated winding already has a clear meaning in electrical machines, i.e. a winding where the number of slots per pole and phases equals one. Examples 3 and 6 are an overlapping concentrated and a non-overlapping concentrated winding respectively.

TABLE III: Winding examples

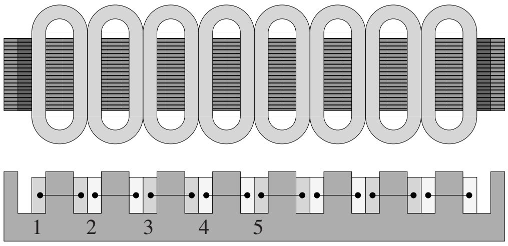
No.	Q_s Q_c $2p$	q q_n/q_d	q_c q_{c_n}/q_{c_d}	y_p, y_d	Classification
1	108 108 10	18/5	18/5	10.8, 10	**Overlapping** fractional pitch fractional slot distributed
2	36 18 4	3/1	3/2	9, 9	**Overlapping** full-pitch integral slot distributed
3	54 27 18	1/1	1/2	3, 3	**Overlapping** full-pitch integral slot concentrated
4	21 21 26	7/26	7/26	0.81, 1	**Non-overlapping** fractional pitch fractional slot distributed
5	72 36 68	6/17	3/17	1.06, 1	**Non-overlapping** fractional-pitch fractional slot distributed
6	30 15 20	1/2	1/4	1.5, 1	**Non-overlapping** fractional-pitch fractional slot concentrated

V. Integral and fractional coil windings

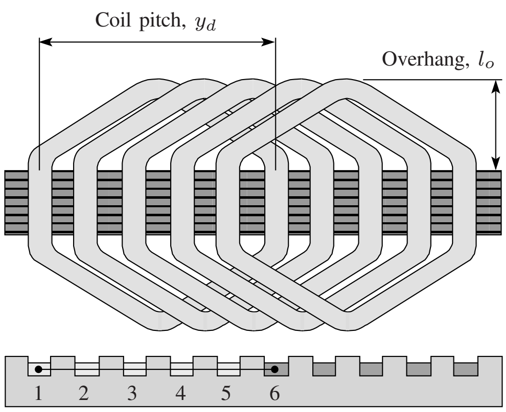
The authors conclude the paper with a new idea, which is found to be useful when comparing non-overlapping windings. According to the classification of the three-phase windings in Fig. 2, non-overlapping windings are always fractional slot



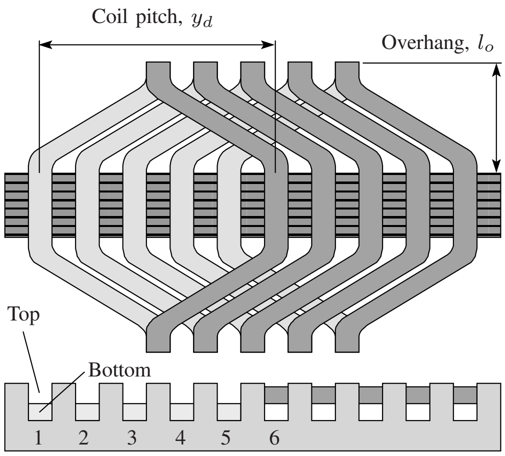
(a) Non-overlapping single layer



(b) Non-overlapping double layer



(c) Overlapping single layer



(d) Overlapping double layer

Fig. 3: Winding types

windings, except in some rare, unused cases. Therefore, non-overlapping windings are practically never integral slot windings. However, when comparing layouts of non-overlapping and overlapping windings the authors found that it is very useful to define *integral-* and *fractional coil windings* as presented in [1]. This is explained in the following sections.

A. *Definition*

Equations (1) and (2) are used to define the number of coil phase groups q_{cb} in a coil phase belt. In this way, integral coil and fractional coil windings are defined as

$$q_{cb} \begin{cases} = 1 & \text{integral coil} \\ > 1 & \text{fractional coil} \end{cases} \tag{12}$$

where

$$q_{cb} = \begin{cases} q_d & \text{for overlapping} \\ 3\, q_{c_n} \dfrac{Q_s}{Q_c} \left| 1 - \dfrac{2p}{Q_s} \right| & \text{for non-overlapping} \end{cases} \tag{13}$$

B. *Number of coils in a phase group*

The *basic winding* is the smallest repetitive winding segment. However, in FE-modelling the *basic winding* can in some cases be divided into smaller sections based on the symmetry conditions. The latter will depend on the number of coils in a phase group. This results in a smaller FE-model with a reduced solution time. The number of coils distributed in a phase group is the numerator q_{c_n} as defined in (2). It can also be defined in terms of the number of sections in the basic winding W_s, i.e.

$$q_{c_n} = \frac{Q_c}{m\, W_s} \qquad \begin{cases} W_s = \gcd(2p, Q_c) \\ m \text{ is the number of phases} \\ q_{c_n} \in \mathbb{N} \end{cases} \tag{14}$$

which must be an integer number for the winding to be valid. In the case of concentrated non-overlapping windings, there is only one coil in a coil phase group ($q_{c_n} = 1$). For distributed non-overlapping windings there are two or more coils ($q_{c_n} = 2, 3, \ldots$) distributed and connected into series to form a coil phase group.

C. *Examples*

The definition of the number of coil groups in a coil phase belt is applied to some of the examples given in Table III. Let us take the examples of two double layer windings; one is an overlapping winding and the other a non-overlapping winding. The results are presented in Table IV as 1b (5 coil groupings and 18 coils) and 4b (5 coil groupings and 7 coils).

Let us consider two single layer windings; one is an overlapping winding and the other a non-overlapping winding. In Table IV these are denoted as 2b (1 coil group and 3 coils) and 5b (1 coil group and 3 coils).

Note that q_{cb} in the case of the non-overlapping winding gives more information about the non-overlapping winding layout, namely if $q_{cb} = 1$ then all the q_{c_n} coils in a coil phase belt are grouped together, while if $q_{cb} > 1$ then all the q_{c_n} coils are distributed in an unsymmetrical way in the coil phase belt; the former is a typical characteristic of a fractional slot winding.

TABLE IV: Winding examples – part 2

No.	Q_s Q_c $2p$	q_d	q_{c_n}	q_{cb}	Classification
1b	108 108 10	5	18	5	**Overlapping** fractional pitch fractional slot distributed *fractional coil*
4b	21 21 26	26	7	5	**Non-overlapping** fractional pitch fractional slot distributed *fractional coil*
2b	36 18 4	1	3	1	**Overlapping** full-pitch integral slot distributed *integral coil*
5b	72 36 68	17	3	1	**Non-overlapping** fractional pitch fractional slot distributed *integral coil*

D. Periodical boundary conditions for FE modelling

Note that q_{c_d} from (2) gives the (minimum) number of poles necessary that have to be meshed in the FE modelling of the machine, and, as explained by [1], if q_{c_d} is uneven then negative periodical boundary conditions must be applied in the FE analysis. If this is not done, then if q_{c_d} is even then positive periodical boundary conditions must be used.

VI. Conclusion

The paper uses well-known winding definitions and terminology and from this presents a classification scheme to relate different winding types to each other. It is also a useful guideline to compare windings that belong to the same category. The unique contribution of the paper is to point out that non-overlapping windings can be concentrated or distributed. Furthermore, the authors introduce a new concept of integral- and fractional coil windings for better comparison of non-overlapping windings.

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